Security Constrained AC Dynamic Transmission Expansion Planning Considering Reactive Power Requirements

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***Abstract*—The Transmission Network Expansion Planning problem (TNEP) can be modeled either as a static, a pseudo-dynamic, or a dynamic problem. Most of the existing formulations do not include reactive power planning within the TNEP problem, leading to sub-optimal designs leading to higher system costs in reality. This paper proposes a dynamic (multi-stage) non-convex formulation that optimizes the addition of transmission circuits and reactive power compensation devices, accounting for operational costs including losses. The planning is done considering *(N-1)* security constraints using an AC model. As there are no similar research works to benchmark the outcomes, the results were compared with those obtained from the static and pseudo-dynamic approaches, showing that the proposed approach provides more economical solutions. It is also shown that better solutions are obtained when Reactive Power Planning (RPP) is considered in the problem formulation. An improved Differential Evolution (DE) and Continuous Population Based Incremental Learning (PBILc) hybrid solution method (IDE-PBILc) is proposed which drastically improves calculation time and robustness. Comparisons with two different state-of-the-art metaheuristics are performed for validation. The results were obtained for the Garver 6-bus, IEEE 24-bus, and the IEEE 118-bus systems. Even though in this work uncertainties are not considered, the proposed approach could be of particular use when studying systems with high renewable energy penetration scenarios, due to its computational efficiency.**

***Index Terms*—Dynamic Transmission Expansion Planning, Non-Convex Optimization, Reactive Power Planning, Renewable Energy, Metaheuristics.**

NOMENCLATURE

Indices and Sets

D	Index of demand.
G	Index of generating unit.
k, l	Index of bus.
i	Index of individual (metaheuristic).
j	Index of the dimension of the individual (metaheuristic).
Ω	Set of all rights-of-ways.
$\wedge$	Set of all load nodes.
Υ	Set of all right-of-ways where a transmission circuit was added.
ϱ	Set of all right-of-ways where were not added transmission circuits.
ξ	Set randomly (20% of the total right-of-ways).
$\mathbb{Z}$	Set of integers.

Functions

v	Cost of transmission circuits, operation, shunt compensation and losses (O. F.) for all stage t.
c_{loss}^t	Total cost of power losses in stage t.
c_{op}^t	Total operation cost in stage t.
w^t	Costs of both not served active and reactive power.

Parameters

T_F	Number of expansion stages or years.
c_{kl}^t	Cost of circuits added between nodes k and l in stage t.
n_{kl}^t	Number of circuits added between nodes k and l in stage t.
$\overline{n}_{kl}^t$	Maximum number of circuits between nodes k and l in stage t.
$\underline{n}_{kl}^t$	Minimum number of circuits between nodes k and l in stage t.
d	Annual discount rate.
α_1^t	Cost of the non-supplied active power in the stage t.
α_2^t	Cost of the non-supplied reactive power in the stage t.
P_D^t	Active power demand. in stage t.
Q_D^t	Reactive power demand. in stage t.
$\underline{P}_G^t/\overline{P}_G^t$	Minimum/Maximum active power generation of unit. G in stage t.
$\underline{Q}_G^t/\overline{Q}_G^t$	Minimum/Maximum reactive power generation of unit. G in stage t.
$\underline{V}/\overline{V}$	Minimum/Maximum voltage magnitude in stage t.
h	Number of hours in a year (8760).
λ^t	Unit cost of the energy for each stage t (\$/MWh).
f_{loss}^t	Factor of losses for each stage t.
β_k	Cost per generation ($\$/MWh$) at node k
γ_k	Cost per generation ($\$/MVArh$) at node k
CF_k	Capacity factor of the generator at node k
$\overline{S}$	Limit of apparent power flows in stage t.
φ_2	Penalty constant when the topology does not converge (10^{15}).
φ_1	Penalty constant when the topology use fictitious

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	generators (10^7).
p_2	Penalty factor.

Variables

n_{kl}^t	Number of circuits added between nodes k and l in the stage t.
r_P^t	Non-supplied active power in the stage t.
r_Q^t	Non-supplied reactive power in the stage t.
V^t	Voltage magnitudes in stage t.
θ^t	Phase angle vector in stage t.
P_G^t	Active power generation vectors in stage t.
Q_G^t	Reactive power generation vectors in stage t.
$S^{from,t}/S^{to,t}$	Apparent power flows (MVA) through the branches in both terminals in stage t.

Metaheuristics Variables

N_I	Size of population or total number of individuals.
n	Size of the individual.
I_{max}	Max. number of iterations.
$F \in [0,2]$	Mutation rate .
$C_r \in [0,2]$	Crossover factor.
η	Learning rate.
$N_{best} \in [0, N_I]$	Number of individuals with the best solutions.
μ^0, σ^0	Mean and standard deviation.
p_{comb}	Combination probability.
$p_{double-mut}$	Probability of mutation.
χ	Constriction factor.
k_1 and k_2	Cognitive and social parameter.
NR	Neighbourhood radius.
x_i	Individual i.
u_{best}	Modified individual.

I. INTRODUCTION

The Transmission Network Expansion Planning (TNEP) problem aims to identify the needs for new transmission assets in time and place for a particular transmission system under different future scenarios. The objective is to meet the future demand optimally while satisfying a set of economic and technical constraints in both normal and contingency conditions [1].

The TNEP is modeled as a combinatorial mixed-integer, non-convex, non-linear problem, which is very difficult to solve (due to the non-linear and non-convex nature) [2], [3]. Due to this complexity, many works rely on linearized models, where the TNEP is modeled as a Mixed Integer Linear Programming (MILP) problem. Linearized models, particularly the DC model, have been widely studied in the literature [4]–[7]. However, research works have shown that the use of simplified models can yield results that are far from reality [3], [8], [9].

TNEP can be classified as either static or dynamic [10], [11]. In the single-stage Static Transmission Expansion Planning (STEP) approach, the grid planner determines the optimal location, size and type of grid expansion for a given planning horizon. STEP has been extensively studied using both simplified models [4]–[7] and complete models [3], [9], [12]–[16]. In the Dynamic Transmission Expansion Planning (DTEP) approach, the planning horizon is divided into several stages, and the planner must also determine the moment (*when*) at which the new infrastructure investments should be made [1], [17].

The specialized literature proposes various methodologies to solve the DTEP problem. Reference [17] proposes a linear disjunctive model to solve the DTEP problem considering security constraints. In [18]–[22], a DC model is used, whereas [20] considers the operating costs and [19], [23] include a reliability criterion. Linearized models to solve the DTEP are presented in [1], [11], [24], where only [11] deals with RPP. In [25], a linearized AC optimal power flow formulation is used to solve the DTEP problem in a two-stage framework ensuring user-defined reliability levels. The authors of [26] present a two-stage strategy to solve the DTEP problem using an AC model considering the N-1 contingency criterion. In [27] a mixed-integer linear programming model is combined with an AC optimal power flow model to solve the DTEP problem, where a two-stage optimization strategy (first DC and then AC) for solving the DTEP problem, taking into account the reactive expansion planning and (N-1) security constraints, is proposed. In [28] a convex model is used for solving the DTEP problem considering the reactive expansion planning. In [29] the AC model is used for solving the DTEP where the investment cost, the total costs of energy losses and RPP are considered. In [30] the AC model is used to solve DTEP problem, considering a multi-voltage approach, power losses and RPP. Finally, in [29] the generation and transmission expansion planning considering switched capacitor bank allocation and demand response program problem is proposed using the AC model.

DTEP is more challenging to solve than STEP, since incorporating several stages results in more decision variables. To reduce the difficulties associated with DTEP, some approaches solve DTEP using a sequence of static sub-problems (known as pseudo-dynamic or quasi-dynamic approaches) named here as Quasi Dynamic Transmission Expansion Planning (QTEP). A forward/backward approach is applied in [31] to solve QTEP using a simplified model. The main disadvantage of QTEP is that the solutions obtained can be far from optimal in some cases, leading to an over-investment in the electric network.

The solution method's performance for TNEP depends on the chosen formulation. TNEP is a NP-hard problem for which there are currently no mathematical solvers are available, capable of solving the TNEP problem in a reasonable time for medium or large grids. However, some classic optimization techniques have been proposed, as in [1], [17], [24], using simplified models. However, these approaches still have computational issues which make them applicable to only the smallest, not realistic problems, or need to resort to convex relaxations which lack accuracy if the convex hull cannot be specified in a sufficiently tight manner [32].

Metaheuristic techniques allow to solve the TNEP problem with more complex formulations. Metaheuristic techniques such as Ant Colony Optimization (ACO) [38], Genetic Algorithm (GA) [18], [26], Hybrid Genetic Algorithm (HGA) [21], Particle Swarm Optimization (PSO) [19], Chaos Particle Swarm Optimization (CPSO) [10], Adaptive Particle Swarm Optimization (APSO) algorithm [22], Hybrid Differential Evolution Algorithm (named LSHADE-SPA) [23], combination of firefly algorithm and harmonic search algorithm (FF+HSA)

Table I

Different approaches to solve the DTEP.

Reference	DTEP (# of stages)	System (# candidate lines)	Model	RPP	Security constraints	Operating costs	Comparison of solution techniques	Search space	Optimality	Convergence
[17]	✓(3)	Colombian 93-bus (10)	Linear disjuntive	×	✓	×	×	Small	Non-Guaranteed	Non-Guaranteed
[18]	✓(2)	Brazilian North-Northeast(-)	DC	×	×	×	×	-	Non-Guaranteed	Non-Guaranteed
[19]	✓(10)	24-bus (34)	DC	×	✓	×	✓	Small	Non-Guaranteed	Non-Guaranteed
[20]	✓(4)	24-bus (28)	DC	×	✓	✓	×	Small	Non-Guaranteed	Non-Guaranteed
[21]	✓(3)	Colombian 93-bus (28)	DC	×	×	×	×	Medium	Non-Guaranteed	Non-Guaranteed
[23]	✓(5)	WDN (31)	DC	×	✓	×	×	Small	Non-Guaranteed	Non-Guaranteed
[11]	✓(3)	118-bus (186)	Linealizad AC	✓	✓	✓	×	Large	Non-Guaranteed	Non-Guaranteed
[25]	✓(10)	118(186)	Linealizad AC	×	×	×	×	Large	Non-Guaranteed	Non-Guaranteed
[14]	✓(3)	46-bus (158)	Convex AC	✓	×	×	✓	Medium	Guaranteed	Non-Guaranteed
[28]	✓(3)	46-bus (158)	Convex AC	✓	×	×	✓	Medium	Guaranteed	Non-Guaranteed
[33]	✓(6)	24-bus (-)	Non-convex AC	×	×	×	✓	Small	Non-Guaranteed	Guaranteed
[26]	✓(10)	118-bus (186)	Non-convex AC	×	✓	×	✓	Large	Non-Guaranteed	Guaranteed
[34]	✓(3)	6-bus (15)	Non-convex AC	✓	✓	×	×	Small	Non-Guaranteed	Guaranteed
[35]	✓(10)	118-bus (20)	Non-convex AC	×	×	×	✓	Small	Non-Guaranteed	Guaranteed
[29]	✓(3)	WDN (32)	Non-convex AC	✓	×	✓	✓	Small	Non-Guaranteed	Guaranteed
[36]	✓(3)	118-bus (186)	Non-convex AC	✓	×	✓	×	Large	Non-Guaranteed	Guaranteed
[37]	✓(3)	118-bus (186)	Non-convex AC	✓	✓	×	×	Large	Non-Guaranteed	Guaranteed
[30]	✓(5)	6-bus (15)	Non-convex AC	✓	×	✓	×	Small	Non-Guaranteed	Guaranteed
Proposed approach	✓(10)	118-bus (186)	Non-convex AC	✓	✓	✓	✓	Large	Non-Guaranteed	Guaranteed

[36], improved binary bat algorithm (IBBA) [29], have shown their potential to solve the DTEP problem, finding high quality and AC feasible solutions (not necessarily optimal) within an acceptable computing time, even for large-scale problems [18], [26]. Therefore, the DTEP is solved in this research work using an improved version of the powerful hybrid metaheuristic known as DE-PBILc. DE-PBILc, an awarded metaheuristic by IEEE PES [39], is based on the combination of two metaheuristics (differential evolution (DE) and continuous population-based incremental learning (PBILc)). Although they cannot guarantee to find globally optimal solutions, AC TNEP models based on metaheuristics converge to high quality solutions [40], [41].

Table I shows a taxonomy of the different approaches to solve the DTEP problem where some of the research gaps are filled in this proposed research work.

A. *Contributions*

From the literature review (see Table I), most research works that propose a DTEP approach use simplified models without taking into account important aspects, in the whole problem, such as reactive power expansion, contingencies, losses, etc. Therefore, this research work presents the following contributions.

1) The formulation of a comprehensive non-linear, non-convex DTEP model to jointly optimize the transmission grid expansion and reactive power compensation investments, including security constraints in the planning problem.
2) A novel meta-heuristic solution technique using an improved differential evolution and continuous population-based incremental learning (IDE-PBILc) approach.
3) The demonstration of the performance of the developed solution technique through rigorous comparison with several metaheuristic solution techniques for static and dynamic TNEP problems.

The results are demonstrated on the Garver 6-node system, the IEEE 24-node system, and the IEEE 118-node system for high renewable energy penetration scenarios.

II. MATHEMATICAL FORMULATION

The mathematical model of DTEP is a hierarchical, bi-level formulation divided into two problems: the expansion master problem and the operational problem, as considered in [3] for the STEP. The expansion planning problem, solved by the IDE-PBILc optimization technique, minimizes the investment cost in transmission circuits, shunt compensation devices, and operational costs, including losses and load shedding, at the end of the planning horizon. The problem formulation also considers the $(N-1)$ security criterion. The transmission expansion master problem is given by

$$\min \ v = \sum_{(t=1)}^{T_F} \left(\frac{\left(\sum_{(k,l)\in\Omega} c_{kl}^t n_{kl}^t + c_{op}^t + c_{loss}^t\right)}{(1+d)^{t-1}} + w^t \right) \quad (1)$$

$$s.t$$

$$\underline{n_{kl}^t} \leq n_{kl}^t \leq \overline{n_{kl}^t}; \quad (n_{kl} \text{ integer}) \quad (2)$$

Since the planning horizon is divided into time stages (t), the cost of operation, circuit additions, and losses for each stage t must be discounted to its present value using a discount rate d. w^t, calculated by (3), corresponds to the costs of both non-supplied active and reactive powers, under the base case and $(N-1)$ contingency conditions (from a list of contingencies), in each stage t.

$$w^t = \sum_{(m=0)}^{nl} w^{m,t} + n_c^t p_2^{m,t} \quad (3)$$

where $m = 0$ corresponds to the normal operation condition (base case) for each state t and $m = 1, \ldots, nl$ corresponds to the system with single circuit contingencies in each stage t. The cost of non-supplied active and reactive power is determined using the optimal power flow model presented in (4)-(13) which minimizes the amount of non-supplied active and reactive power w^t for a given time step t and contingency m. In the optimization model, the non-supplied active and reactive power are modeled using fictitious generators the $r_{P_k}^{m,t}$ and $r_{Q_k}^{m,t}$. The corresponding optimal power flow model is nonlinear and nonconvex and can be efficiently solved using interior-point solvers.

$$\min \ w^{m,t} = \sum_{(k\in\wedge)} (\alpha_1^{m,t} r_{P_k}^{m,t} \varphi_1) + \sum_{(k\in\wedge)} \left(\frac{\alpha_2^{m,t} r_{Q_k}^{m,t}}{(1+d)^{t-1}} \right) \tag{4}$$

$s.t$

$$\boldsymbol{P}(\boldsymbol{V},\boldsymbol{\theta})^{m,t} - \boldsymbol{P}_G^{m,t} + \boldsymbol{P}_D^{m,t} - \boldsymbol{r}_P^{m,t} = 0 \tag{5}$$

$$\boldsymbol{Q}(\boldsymbol{V},\boldsymbol{\theta})^{m,t} - \boldsymbol{Q}_G^{m,t} + \boldsymbol{Q}_D^{m,t} - \boldsymbol{r}_Q^{m,t} - \boldsymbol{r}_Q^{m,t-1} = 0 \tag{6}$$

$$\underline{\boldsymbol{P}}_G^t \leq \boldsymbol{P}_G^{m,t} \leq \overline{\boldsymbol{P}}_G^t \tag{7}$$

$$\underline{\boldsymbol{Q}}_G^t \leq \boldsymbol{Q}_G^{m,t} \leq \overline{\boldsymbol{Q}}_G^t \tag{8}$$

$$\underline{\boldsymbol{r}}_P^t \leq \boldsymbol{r}_P^{m,t} \leq \overline{\boldsymbol{r}}_P^t \tag{9}$$

$$\underline{\boldsymbol{r}}_Q^t \leq \boldsymbol{r}_Q^{m,t} \leq \overline{\boldsymbol{r}}_Q^t \tag{10}$$

$$\underline{\boldsymbol{V}}^t \leq \boldsymbol{V}^{m,t} \leq \overline{\boldsymbol{V}}^t \tag{11}$$

$$\boldsymbol{S}^{from,m,t} \leq \overline{\boldsymbol{S}} \tag{12}$$

$$\boldsymbol{S}^{to,m,t} \leq \overline{\boldsymbol{S}} \tag{13}$$

1) (N-1) Security Criterion: For cases where no feasible solution of the optimization model is obtained, the term $n_c^t p_2^{m,t}$ is introduced in (3), where n_c^t corresponds to the number of times that the fictitious generator is utilized within the set of nl contingencies. p_2 is a high penalty factor ($p_2 = \varphi_2$) when no feasible solution to the optimal power flow model is obtained, and $p_2 = 0$ otherwise. Using this approach, network topologies where fictitious generators need to be utilized to find a feasible solution to the operational problem become less attractive compared to those using only existing generation ($w = 0$). Algorithm 1 outlines the detailed methodology for penalizing contingencies leading to non-feasible system states. Note that in this work a predefined set of critical line contingencies have been used, which remains the same throughout all time points.

Algorithm 1 Flowchart for base case and contingency penalization strategy.

```
$m = 0$ (base case) and $nl$ (number of contingencies)
for $t = 1 : T_F$ do
    $n_c^t = 1$
    for $m = 0 : nl$ do
        $p_1^{m,t} = p_2^{m,t} = 0$
        Evaluate Topology. Eq. (4)-(13)
        if Eq. (4)-(13) converge then
            $p_2^{m,t} = 0$
            if Fictitious Gen.> 0 then
                $p_1^{m,t} = \sum_{(k\in\wedge)} (\alpha_1 \cdot r_{pk}^{m,t}) \cdot \varphi_1$
                $n_c^t = n_c^t + 1$
            else
                $p_1^{m,t} = 0$
            end if
        else
            $p_2^{m,t} = \varphi_2$
            $n_c^t = 1$
        end if
    end for
end for
Find Eq. (3)
```

*$p_1^{m,t}$, $p_2^{m,t}$ Penalty constants

2) Reactive power compensation requirements: In the proposed formulation, the shunt compensation is modeled through fictitious reactive power generators ($\boldsymbol{r}_Q^t$), for each stage t. The term $(\alpha_2^t r_{Q_k}^t / (1+d)^{t-1})$ discounted back to its present value) represents the costs for both the required capacitive and inductive reactive power compensation equipment. In order to maintain a positive cost for compensation, the cost coefficient α_2^t is defined to be positive for positive reactive power provision (capacitive compensation) and negative for negative reactive power provision (inductive compensation). A detailed explanation of reactive power compensation model can be found in [3].

Since the formulation of the proposed DTEP model considers contingency conditions, the necessary reactive compensation at each node k is the maximum capacitive compensation and the minimum inductive compensation allowing feasible operating conditions among all considered contingencies. The required reactive power compensation is represented mathematically by

$$\begin{aligned} r_{Q_{k\in\wedge}}^{m,t} &= \max(r_{Q_{k,C}}^{0,t}, r_{Q_{k,C}}^{1,t}, ..., r_{Q_{k,C}}^{nl,t}) \\ &+ \min(r_{Q_{k,L}}^{0,t}, r_{Q_{k,L}}^{1,t}, ..., r_{Q_{k,L}}^{nl,t}), \end{aligned} \tag{14}$$

where $r_{Q_{k,C}}^{0,t}$ and $r_{Q_{k,L}}^{0,t}$ represent the capacitive and inductive reactive power generated by the fictitious generator at node k for the base case ($m = 0$) at each stage t, respectively; $r_{Q_{k,C}}^{m>0,t}$ and $r_{Q_{k,L}}^{m>0,t}$ represent the capacitive and inductive reactive power generated at node k for nl contingencies ($m > 0$) at each stage t, respectively.

3) Operational cost: The operational cost (c_{op}) is associated with the cost of power generation to satisfy demand. The operational cost for stage t is represented by

$$c_{op}^t = h \cdot \sum_{(k)\in\wedge} \left(\frac{\beta_k^t \cdot P_{Gk}^t \cdot CF_k + \gamma_k^t \cdot Q_{Gk}^t}{(1+d)^{t-1}} \right) \tag{15}$$

In this paper, the optimization of the operational cost of reactive power generation is not considered (i.e., $\gamma_k = 0$).

4) Transmission losses: Although the generation implicitly considers transmission losses, defining a separate cost term for losses allows to evaluate the influence of active power losses in the final transmission plan. In this paper, the total annual loss cost for any stage t is approximated by

$$c_{loss}^t = h \cdot \lambda^t \cdot f_{losses}^t \cdot P_{total}^t \tag{16}$$

where λ^t corresponds to the losses cost in the stage t, f_{loss}^t is the relation between the average loss power in a year and the loss power that occurs with the maximum demand in that year, and P_{total}^t corresponds to the total active power losses of a topology for the stage t, which is given by

$$P_{total}^t = \sum_{(k,l)\in\Omega} P_{kl}(\boldsymbol{V},\boldsymbol{\theta})^t + P_{lk}(\boldsymbol{V},\boldsymbol{\theta})^t \tag{17}$$

5) Demand and generation growth: The growth of both demand and generation is approximated by discrete increases (annual load growth Δ) added for each dynamic stage.

III. IMPROVED DE-PBILC (IDE-PBILC) OPTIMIZATION ALGORITHM

To solve the DTEP problem, the hybrid metaheuristic optimization technique (DE-PBILc) based on the combination of Continuous Population-Based Incremental Learning (PBILc) and the Differential Evolution (DE) metaheuristics, was used

[42]. In each iteration k, the population is composed of N_I individuals $\boldsymbol{x}^k = [x_1^k, ..., x_i^k, ..., x_{N_I}^k]$, where each individual i is an n-dimensional vector $x_i^k = [x_{i,1}^k, x_{i,2}^k, ..., x_{i,n}^k]$ represents a candidate solution (candidate topology). To create new solutions, each individual is generated using Differential Evolution (DE) or PBILc, subsequently, the parameters of the metaheuristic are updated based on the new solutions. For the deatils of the (DE-PBILc) algorithm the readers are referred to [42].

In this work, DE-PBILc is improved by combining various techniques (Random Search (RS), Chaos Theory (CT), and a Local Search technique (LS1)) to generate new individuals (new population) to avoid convergence to possible local optima (see Algorithm 2). LS1 identifies the circuits that have been added to the system, and using an iterative process, each added transmission circuit is eliminated. Once a new population is generated, the new individuals compete with the individuals of the previous generation.

Algorithm 2 Pseudo-code of Random Search (RS), Chaos Theory (CT)) and Local Search technique (LS1).

```
 1: $x_{rand}$ (Generate a randomly number $\epsilon[0,1]$.)
 2: for $i = 1 : N_I$ do
 3:     if $0 < x_{rand} <= 0.3$ then  Apply random search (RS)
 4:         Step:$\xi$ (Select randomly a 20% of the total dimension individual $i$)
 5:         for $j = 1 : size(\xi)$ do
                $u_{i,\xi_j}^k = x_{i,\xi_j}^{k-1} + (x_{i,\xi_j}^{k-1} - x_{i,\xi_j}^{k-1}) * rand$     (18)
 6:         end for
 7:     else if $0.3 < x_{rand} <= 0.6$ then  Apply Chaos Theory (CT)
 8:         Step: $\xi$ (Select randomly a 20% of the individual $i$)
 9:         for $j = 1 : size(\xi)$ do
                $u_{i,\xi_j}^k = \chi_{CT} \cdot x_{i,\xi_j}^{k-1} \cdot (1 - x_{i,\xi_j}^{k-1})$     (19)
10:         end for
11:     else $x_{rand} > 0.6$  Apply Local Search (LS1)
12:         Step: $\Upsilon$ (Identify where transmission circuits were added in $x_i$)
13:         Step: $\zeta$ (Select a 20% of $\Upsilon$)
14:         for $j = 1 : size(\zeta)$ do
                $u_{i,\zeta_j}^k = x_{i,\zeta_j}^{k-1} - 1$     (20)
15:         end for
16:     end if
17:     Apply Selection: Compare the vector $u_i^k$ with the current population $x_i^k$
18:     if $f(u_i) < f(x_i)$ then $x_i^{k+1} = u_i^k$
19:     else $x_i^{k+1} = x_i^k$
20:     end if
21: end for
```

Additionally, if the value of the objective function does not change after $N_{changes}$ iterations, the Algorithm 3 is applied. Local Search (LS2) identifies the number of transmission circuits added to the best current topology and using an iterative process, each added transmission circuit is eliminated. When each transmission circuit is eliminated, another circuit is added in a different right-of-way. The added transmission circuit must meet the condition that its investment value is less than the investment value of the eliminated transmission circuit. Otherwise, the transmission circuit is not added to that right-of-way. If during this iterative process a better objective function value is obtained, this topology replaces the previous one as the best current solution.

The relationship between the metaheuristic's decision variable and n_{kl} in (1) is represented in (21), where x_{min} is the initial topology of the system.

$$n_{kl}^t = \begin{cases} x_{i,j}^t - x_{min,1,j} & if \quad t = 1 \\ x_{i,j}^t - x_{i,j}^{t-1} & if \quad t > 1 \end{cases} \tag{21}$$

Algorithm 3 Pseudocode for LS2.

```
 1: Step: $c_{count} = N_{changes}$
 2: Step: $x_{best}^k$ (Identify the individual with the best value of O.F.)
 3: Step: $\Upsilon$ (Identify where transmission circuits were added in $x_{best}^k$.)
 4: Step: $\varrho$ (Identify where transmission circuits were not added in $x_{best}^k$.)
 5: for $i = 1 : size(\Upsilon)$ do
            $u_{best,i}^k = x_{best,1,\Upsilon_i}^k - 1$     (22)
 6:     for $j = 1 : size(\varrho)$ do
 7:         if $i \neq j$ and $c_{\varrho_j} < c_{\Upsilon_i}$ then
                $u_{best,\varrho_j}^k = u_{1,\varrho_j}^k + 1$     (23)
 8:             Step: Selection. Similar to Algorithm 2 (Step 17)
 9:         end if
10:     end for
11: end for
```

*c_{Υ_i} corresponds to the transmission circuit cost to deleted , c_{ϱ_j} corresponds to the transmission circuit cost to added, and c_{count} corresponds to a counter of the number of times the O.F. does not change.

IV. DTEP IMPLEMENTATION USING IDE-PBILc

Algorithm 4 shows the pseudo-code of the IDE-PBILc metaheuristic implementation to solve the DTEP problem.

1) Network data: The technical and economic data of the system are uploaded. The technical data is associated with the initial topology of the system (current demand, currently active and reactive power generation, and capacity of existing and candidate transmission circuits). The economic data contains the investment costs for new transmission circuits, reactive power compensation cost, generation and losses costs and operating conditions.

2) Metaheuristics parameters tuning: In addition to the IDE-PBILc, the DE-PBILc [42] and the Local Particle Swarm Optimization (LPSO [3]) were used to validate the results and robustness of the proposed approach. Table II shows the parameters used by the different metaheuristics. Each set of parameters represents the best performing combination for the individual metaheuristic, which is used for all scenarios and test systems. The parameter tuning was performed by a trial and error process using only the Garver 6-bus system. This process is performed previous to the simulations, therefore, this does not affect the computing time. The same set of parameters is used for all the tests and modifications during the simulations are not needed.

3) Initial solutions: The quality of initial solutions impacts the performance of the optimization technique. There are several ways to generate initial solutions for metaheuristic optimization techniques. In this research work, the initial solutions generated for TNEP (STEP, QTEP and DTEP) is based on [43]. For benchmarking purposes, the same set of

Algorithm 4 IDE-PBILc pseudo-code for solving the DTEP.

```
1:  Network Data (Each Stage)
2:  Parameters of the Metaheuristic (Each Stage)
3:  $C_{count} = 1$
4:  Initial Population (Each Stage)
5:  for $i = 1 : N_I$ do
6:      Solve AC Power Flow for $m = 0 : nl$ (see Algorithm 1 )
7:  end for
8:  Calculate O.F. Eq (1)
9:  for $k = 1 : iter_{max}$ do
10:     for $i = 1 : N_I$ do
11:         for $t = 1 : T_F$ do
12:             Generate New Topology or Individual (Algorithm 2)
13:         end for
14:     end for
15:     Apply step 5-8
16:     for $i = 1 : N_I$ do
17:         Apply selection (Algorithm 2, Step 17)
18:     end for
19:     for $i = 1 : N_I$ do
20:         for $t = 1 : T_F$ do
21:             Generate New Topology or Individual (DE-PBILc)
22:         end for
23:     end for
24:     Apply step 5-8
25:     for $i = 1 : N_I$ do
26:         Apply selection (Algorithm 2, Step 17)
27:     end for
28:     if $f(x_{best}^{k}) < f(x_{best}^{k-1})$ then $C_{count} = 1$
29:     else $C_{count} = C_{count} + 1$
30:     end if
31:     if $C_{count} = 20$ then
32:         $C_{count} = 1$
33:         Generate New Topology or Individual for $x_{best}$ (Algorithm 3)
34:         Apply step 5-8 for $i = best$
35:         Apply selection for the individual $x_{best}$ (Algorithm 2, Step 17)
36:     end if
37:     Update Parameters of Each Stage (DE-PBILc) (see [42])
38:     if Stopping Criteria then
39:         END
40:     end if
41: end for
```

Table II

Parameters of the metaheuristics used to solve the DTEP.

Metaheuristic	IDE-PBILc	DE-PBILc	LPSO
Parameters	$F = 1$	$F = 1$	
	$Cr = 0.2$	$Cr = 0.2$	
	$\eta = 0.05$	$\eta = 0.05$	$\chi = 0.729$
	$N_{best} = N_I/2$	$N_{best} = N_I/2$	$k_1 = 2.05$
	μ^0 randomly	μ^0 randomly	$k_2 = 2.05$
	$\sigma^0 = 2$	$\sigma^0 = 2$	$NR = 1$
	$p_{double-mut} = 0.3$	$p_{double-mut} = 0.3$	$v_{max} = 0.5 \cdot \begin{bmatrix} x_{min} \\ x_{max} \end{bmatrix}$
	$p_{comb} = 0.9$	$p_{comb} = 0.9$	
	$\chi_{CT} = rand[1, 2]$		
	$N_{iter} = 20$		

initial solutions was used for each scenario with the different metaheuristics.

The initial solutions are made up of N_I individuals ($x = [x_1, x_2, \ldots x_i, \ldots, x_{N_I}]$), for all time stages, where each individual x_i represents the decision variable of the metaheuristic and corresponds to a transmission topology in the TNEP.

4) Evaluate population or topology: Once the population of N_I individuals is generated for all stages, the value of the objective function for each stage (1) needs to be calculated using the AC-OPF formulation (4)-(13), for which the corresponding value of the operational problem is required.

5) Elimination of repeated topologies: Identifying repeated individuals during the optimization process allows a notable reduction in the computational effort. This process is performed by identifying and saving, in a data matrix, all the individuals (topology and objective function value) of each iteration, in such a way that if in an iteration k a previously generated topology is identified, its objective function will not be evaluated, rather the objective function value previously is restored.

6) Stopping criterion: The simulation process can be stopped when the following criteria are met: *i)* the maximum number of iteration is reached, and *ii)* the value of the objective function does not change for $N_{iter-max}$ iterations.

7) Search space: Suppose that the number of candidate corridors is a n-dimensional vector (which is also the dimension of the problem) and x_{max} is a scalar which represents the maximum number of feasible candidate circuits in each corridor. Assuming that x_{max} is constant for all right-of-ways, the search space for the STEP and QTEP is given by the number of possible topologies $(x_{max} + 1)^n$, while the search space for DTEP is given by $(x_{max}+1)^{n \cdot T_F}$. Evidently, feasible and unfeasible network topologies are included in this calculation.

8) Comparisons criteria: Robustness, e.g., the ability to obtain the same least cost solution for all trials, defined *success rate*.

Optimality, e.g., the least cost solution obtained by a metaheuristic technique.

Computing time, e.g., the required computation time to find the least cost solution. The computing time performance can also be measured by the average number of AC-OPF evaluations (Eval.F.O), or the average number of iterations (Average.Iter.) needed to get the best solution.

V. APPROACH APPLIED TO A 4-BUS EXAMPLE SYSTEM

Fig. 1 shows a 4-bus power system used to explain the core of the DTEP approach using some step-by-step calculations. For the sake of simplicity, contingencies, operational and losses costs are neglected. This example considers $T_F = 3$ dynamic states with an annual discount rate of $d = 4\%$ and an estimated annual growth rate of $\Delta = 6\%$ for both demand and installed generation capacity (active and reactive). The network data is provided in appendix A.

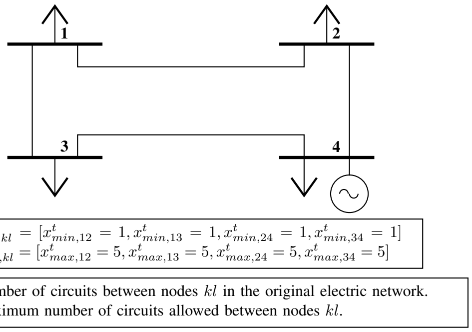

Fig. 1. *Power system example (4-bus).*

The optimization process begins by generating the initial solutions (initial topologies), for which the limits are considered.

For simplicity, only two topologies ($N_I = 2$ individuals) are generated according to the pseudo-random approach explained in [43]. Note that the size of the decision variable array x is $n \cdot T_F$

$$\begin{aligned} x_1^t = [&x^1_{12}=4, x^1_{13}=1, x^1_{24}=1, x^1_{34}=1, x^2_{12}=4, x^2_{13}=1, \\ &x^2_{24}=1, x^2_{34}=3, x^3_{12}=4, x^3_{13}=1, x^3_{24}=1, x^3_{34}=3] \end{aligned} \tag{24}$$

$$\begin{aligned} x_2^t = [&x^1_{12}=1, x^1_{13}=1, x^1_{24}=4, x^1_{34}=1, x^2_{12}=5, x^2_{13}=1, \\ &x^2_{24}=4, x^2_{34}=1, x^3_{12}=5, x^3_{13}=1, x^3_{24}=4, x^3_{34}=1] \end{aligned} \tag{25}$$

Now, each expansion topology for each time stage (t) is evaluated using (4)-(13), obtaining (3).

$$w^t_{x_1} = [w^1_{x_1} = 9 \cdot 10^{12}, w^2_{x_1} = 2 \cdot 10^{12}, w^3_{x_1} = 4 \cdot 10^{12}] \tag{26}$$

Note that the elements of w_{x_1} for topology 1 are much larger than zero because of the penalization for infeasible topologies.

The added circuits for topology 1 are calculated according to (21) as follows:

$$\begin{aligned} n^t_{kl} = [&n^1_{12}=3, n^1_{13}=0, n^1_{24}=0, n^1_{34}=0, n^2_{12}=0, n^2_{13}=0, \\ &n^2_{24}=0, n^2_{34}=2, n^3_{12}=0, n^3_{13}=0, n^3_{24}=0, n^3_{34}=0] \end{aligned} \tag{27}$$

For this example, assuming $c^t_{op} = 0$ and $c^t_{loss} = 0$, without considering contingencies, the objective function (1) is calculated as

$$v = \sum_{(t=1)}^{T_F} \left(\frac{\left(\sum_{(k,l)\in\Omega} c^t_{kl} n^t_{kl}\right)}{(1+d)^{t-1}} + w^t \right) \tag{28}$$

Then, the objective function for topology x_1 using (28) is

$$\begin{aligned} v_{x_1} = [&c^1_{12}n^1_{12}/b + c^1_{13}n^1_{13}/b + c^1_{24}n^1_{24}/b + c^1_{34}n^1_{34}/b + \\ &c^2_{12}n^2_{12}/b + c^2_{13}n^2_{13}/b + c^2_{24}n^2_{24}/b + c^2_{34}n^2_{34}/b + \\ &c^3_{12}n^3_{12}/b + c^3_{13}n^3_{13}/b + c^3_{24}n^3_{24}/b + c^3_{34}n^3_{34}/b + \\ &w^1_{x_1} + w^2_{x_1} + w^3_{x_1}] = 1.6 \cdot 10^{13} \end{aligned} \tag{29}$$

where $b = (1+d)^{t-1}$.

The same process is repeated to find the objective function for the topology x_2 given in (25),

$$w^t_{x_2} = [w^1_{x_2} = 0.4, w^2_{x_2} = 0, w^3_{x_2} = 0] \tag{30}$$

$$\begin{aligned} n^t_{kl} = [&n^1_{12}=0, n^1_{13}=0, n^1_{24}=3, n^1_{34}=0, n^2_{12}=4, n^2_{13}=0, \\ &n^2_{24}=0, n^2_{34}=0, n^3_{12}=0, n^3_{13}=0, n^3_{24}=0, n^3_{34}=0] \end{aligned} \tag{31}$$

$$\begin{aligned} v_{x_2} = [&c^1_{12}n^1_{12}/b + c^1_{13}n^1_{13}/b + c^1_{24}n^1_{24}/b + c^1_{34}n^1_{34}/b + \\ &c^2_{12}n^2_{12}/b + c^2_{13}n^2_{13}/b + c^2_{24}n^2_{24}/b + c^2_{34}n^2_{34}/b + \\ &c^3_{12}n^3_{12}/b + c^3_{13}n^3_{13}/b + c^3_{24}n^3_{24}/b + c^3_{34}n^3_{34}/b + \\ &w^1_{x_2} + w^2_{x_2} + w^3_{x_2}] = 334.2 \end{aligned} \tag{32}$$

The small value $w^1_{x_1} = 0.4$ in (30) is due to the reactive power compensation needed in stage 1 for topology 2.

In a next iteration, IDE-PBILc tries to improve the solutions x_1 and x_2, and the process is repeated until the topology with the best objective function is found. Therefore, at a certain iteration of the optimization process, IDE-PBILc generates its best possible topologies for x_1 and x_2, where the metaheuristic technique is not able to improve the solutions anymore.

$$\begin{aligned} x_1^t = [&x^1_{12}=1, x^1_{13}=1, x^1_{24}=1, x^1_{34}=4, x^2_{12}=1, x^2_{13}=1, \\ &x^2_{24}=2, x^2_{34}=4, x^3_{12}=1, x^3_{13}=1, x^3_{24}=2, x^3_{34}=4] \end{aligned} \tag{33}$$

$$\begin{aligned} x_2^t = [&x^1_{12}=1, x^1_{13}=1, x^1_{24}=2, x^1_{34}=1, x^2_{12}=1, x^2_{13}=1, \\ &x^2_{24}=2, x^2_{34}=2, x^3_{12}=1, x^3_{13}=1, x^3_{24}=2, x^3_{34}=2] \end{aligned} \tag{34}$$

Subsequently, the objective function for the new topology x_1 (33), applying the same procedure as in (26)-(28), is

$$w^t_{x_1} = [w^1_{x_1} = 0.4, w^2_{x_1} = 0, w^3_{x_1} = 0] \tag{35}$$

$$\begin{aligned} n^t_{kl} = [&n^1_{12}=0, n^1_{13}=0, n^1_{24}=0, n^1_{34}=3, n^2_{12}=0, n^2_{13}=0, \\ &n^2_{24}=1, n^2_{34}=0, n^3_{12}=0, n^3_{13}=0, n^3_{24}=0, n^3_{34}=0] \end{aligned} \tag{36}$$

$$v_{x_1} = 118.1 \tag{37}$$

The same process applied to topology 2 (34) gives

$$w^t_{x_2} = [w^1_{x_2} = 0.6, w^2_{x_2} = 0, w^3_{x_2} = 0] \tag{38}$$

$$\begin{aligned} n^t_{kl} = [&n^1_{12}=0, n^1_{13}=0, n^1_{24}=1, n^1_{34}=0, n^2_{12}=0, n^2_{13}=0, \\ &n^2_{24}=0, n^2_{34}=1, n^3_{12}=0, n^3_{13}=0, n^3_{24}=0, n^3_{34}=0] \end{aligned} \tag{39}$$

$$v_{x_2} = 79.8 \tag{40}$$

The best solution found, corresponding to topology 2 (34), presents a total investment of 79.8 M$ with two added circuits ($n^1_{24} = 1$ and $n^2_{34} = 1$) at stage 1 and 2 respectively. Additionally, shunt compensation of 66 MVAr with a total cost of 0.6 M$ is required. The final topology can be seen in Fig. 2.

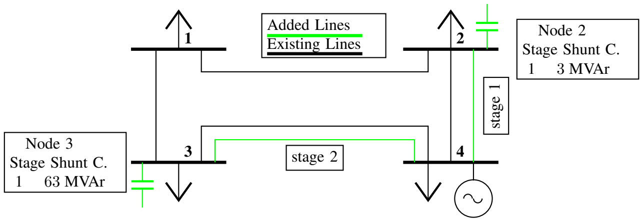

Fig. 2. *Final plan for power system (4-bus).*

VI. RESULTS

This section presents the results of applying the proposed methodology to three test systems: the well-known Garver 6-bus system, IEEE 24-bus system, and the IEEE 118-bus system. The main data for each system such as buses number, candidate branches, and total active and reactive power demand both at the initial stage ($t = 0$) as at the end of the TNEP study ($t = 10$) can be seen in Table III. The developed algorithms were implemented in MATLAB, and calculations have been performed on a PC with an AMD Ryzen 5950x processor with 16 cores, and 64 GB RAM. The simulations considered $T_F = 10$ dynamic states with an annual discount rate of $d = 4\%$. In addition, for the Garver 6-bus system and the IEEE 24-node system, an estimated annual growth rate of $\Delta = 6\%$ for both demand and installed generation capacity (active and reactive) was adopted for each state, while for

Table III

Principal data and search space of the Garver 6-bus, IEEE 24-bus, and the IEEE 118-bus systems.

System	**Garver 6-bus**	**IEEE 24-bus**	**IEEE 118-bus**
# buses	6	24	118
Right-of-way	15	41	186
P_D^0 **(MW)**	424.3	4774	4423
Q_D^0 **(MVAr)**	84.8	971	1751
P_D^{10} **(MW)**	760	8550	6240
Q_D^{10} **(MVAr)**	152	1740	2470
STEP (Search space)	$(5+1)^{15} = 6^{15}$	$(5+1)^{41} = 6^{41}$	$(5+1)^{186} = 6^{186}$
QTEP (Search space)	$(5+1)^{15} = 6^{15}$	$(5+1)^{41} = 6^{41}$	$(5+1)^{186} = 6^{186}$
DTEP (Search space)	$(5+1)^{15\cdot 10} = 6^{150}$	$(5+1)^{41\cdot 10} = 6^{410}$	$(5+1)^{186\cdot 10} = 6^{1860}$
Contingencies (circuits)	l_{1-4}, l_{2-4}, l_{3-5}	l_{2-6}, l_{6-10}, l_{13-23}	l_{17-18}, l_{19-34}, l_{42-49}, l_{59-61}, l_{24-72}

the IEEE 118-bus system, an estimated annual growth rate of $\Delta = 3.5\%$ for both demand and installed generation capacity (active and reactive) was adopted for each state. The solutions of DTEP are compared to STEP [3] and QTEP (forward method [10]). The data for both QTEP and DTEP at the end of the TNEP study match STEP data. The data used to solve the STEP can be found in [3], [12]. The original DE-PBILc version can be downloaded from [39]. IDE-PBILc and all test networks can be requested from authors. The total number of topologies for each system and TNEP problem is also given in Table III. It is worth noting that all transmission paths have been considered for circuits expansion and the maximum allowed number of circuits per right-of-way is chosen to be five for all test cases. Especially for the DTEP problem this results in a very large search space. For each test system, two main scenarios were considered: *i)* Scenario with dispatchable generation without shunt compensation (scenarios A, C, and E), and *ii)* Scenario with dispatchable generation allowing shunt compensation (scenarios B, D, and F with $\alpha_2 = 10$ k\$/MVAr). Each scenario analyzed was considered both without contingencies (A1, B1, C1, D1, E1 and F1) and with contingencies (A2, B2, C2, D2, E2 and F2). The shunt compensation limits were set to -1000 and 1000 MVAr.

Parameters related to the population size, number of trials, and stopping criterion used for the simulations for each system can be seen in Table IV.

Table IV

Additional parameters for the simulations.

System	**Garver 6-bus**			**IEEE 24-bus**			**IEEE 118-bus**		
TEP	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**
Pop. size (N_I)	100	100	100	120	120	120	150	150	150
Trials	10	10	10	10	10	10	5	5	5
I_{max}	1000	1000	1000	2000	2000	2000	3000	3000	3000
$N_{iter-max}$	250	250	250	250	250	250	250	250	250

A. *Individual contributions of the different metaheuristics in the IDE-PBILc*

Fig. 3 shows the individual contributions of RS, CT, LS1 and LS2 to increase the success rate of the DE-PBILc in the DTEP for the Garver 6-bus system. RS, CT, LS1, DE-PBILc and RS-TC-LS1 do not present good success rate. Adding Algorithm 2 to the DE-PBILc (named DE-PBILc-CT-RS-LS1) a success rate of 50% is obtained. Finally, adding Algorithm 3 to the DE-PBILc-CT-RS-LS1 (named IDE-PBILc) allows obtaining a success rate of 70%. These results show that incorporating the different strategies into the DE-PBILc helps to improve its optimality.

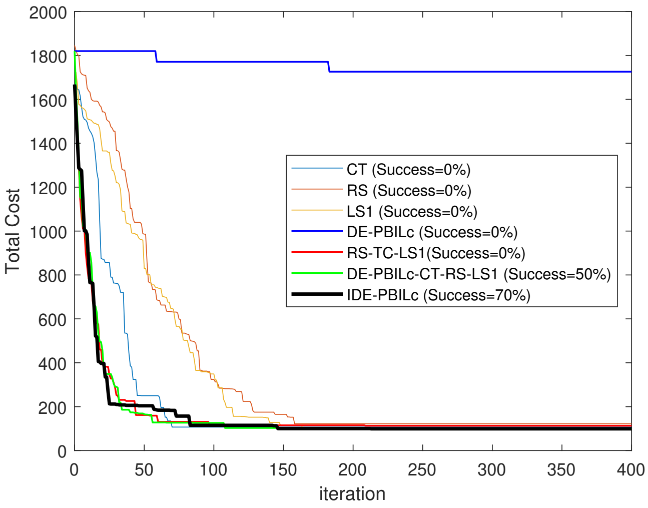

Fig. 3. *Individual contributions of RS, CT, LS1, LS2, DE-PBILc in the IDE-PBILc applied in the DTEP.*

Table V

DTEP results for Garver 6-bus systems (scenarios A1, A2, B1, and B2).

Scenario	**A1**	**B1**	**A2**	**B2**
TEP	**DTEP**	**DTEP**	**DTEP**	**DTEP**
Added Lines$_{(year)}$	$l_{2-6} = 1_{(1)}$ $l_{3-5} = 1_{(1)}$ $l_{4-6} = 1_{(1)}$ $l_{2-6} = 1_{(2)}$ $l_{4-6} = 1_{(5)}$ $l_{3-5} = 1_{(9)}$	$l_{4-6} = 2_{(1)}$ $l_{3-5} = 1_{(6)}$ $l_{2-6} = 1_{(9)}$	$l_{2-6} = 1_{(1)}$ $l_{3-5} = 2_{(1)}$ $l_{4-6} = 2_{(1)}$ $l_{2-6} = 1_{(5)}$ $l_{3-5} = 1_{(9)}$	$l_{2-6} = 1_{(1)}$ $l_{3-5} = 1_{(1)}$ $l_{4-6} = 1_{(1)}$ $l_{3-5} = 1_{(5)}$ $l_{4-6} = 1_{(8)}$
Lines Added	6	4	7	5
Total Cost (M\$)	149.1	99.46	170.2	120.7
Lines Cost (M\$)	149.1	98.35	170.2	119.9
Total Shunt Comp. Cost (M\$)	-	1.08	-	0.8
Total Capacitive Comp. (MVAr)	-	127	-	95
Cap. Comp. (MVAr) $r_{year}(r_{Q,gen})$	-	$32_{(1)}(r_{Q2} = 16, r_{Q5} = 16)$; $10_{(2)}(r_{Q5} = 10)$; $8_{(3)}(r_{Q5} = 8)$; $9_{(4)}(r_{Q5} = 9)$; $18_{(5)}(r_{Q4} = 8, r_{Q5} = 10)$; $5_{(7)}(r_{Q2} = 3, r_{Q4} = 2)$; $46_{(8)}(r_{Q2} = 44, r_{Q4} = 2)$;	-	$17_{(1)}(r_{Q4} = 3, r_{Q5} = 14)$; $6_{(2)}(r_{Q4} = 1, r_{Q5} = 5)$ $10_{(3)}(r_{Q4} = 6, r_{Q5} = 4)$; $23_{(4)}(r_{Q4} = 2, r_{Q5} = 21)$; $3_{(5)}(r_{Q4} = 3)$; $6_{(6)}(r_{Q4} = 6)$; $30_{(7)}(r_{Q4} = 30)$
Saving in invest. by shunt comp. (M\$)		49.6		49.5

B. *Garver 6-bus system*

1) Comparison of scenarios A and B for DTEP: Table V show the results of the DTEP for the Garver system. In general, scenarios B (using shunt compensation) present the possibility of avoiding to build additional transmission circuits by placing shunt compensation in certain buses. Even when scenario B2 includes contingencies, it presents a lower investment cost than scenario A1 (without including contingencies) because of the shunt compensation. Also, results show the investment year (transmission circuits and reactive power compensation) in which the most economical benefits are obtained.

2) Comparison among STEP, QTEP and DTEP (Scenario B1): In Table VI, it is shown the final plan with the best economic benefit is obtained by DTEP if compared to STEP and

Table VI

TEP results for Garver 6-bus systems considering only circuits cost and shunt compensation (scenarios B1).

Scenario	**B1**		
TEP	**STEP**	**QTEP**	**DTEP**
Added Lines$_{(year)}$	$l_{2-6}=1$ $l_{3-5}=1$ $l_{4-6}=2$	$l_{2-6}=1_{(1)}$ $l_{4-6}=1_{(1)}$ $l_{3-5}=1_{(5)}$ $l_{3-5}=1_{(9)}$ $l_{4-6}=1_{(10)}$	$l_{4-6}=2_{(1)}$ $l_{3-5}=1_{(6)}$ $l_{2-6}=1_{(9)}$
Lines Added	4	5	4
Total Cost (M$)	110.44	114.33	99.46
Lines Cost (M$)	110	112.78	98.35
Total Shunt Comp. Cost (M$)	0.44	1.52	1.08
Total Capacitive Comp. (MVAr)	44	195	127
Cap. Comp. (MVAr) $r_{year}(r_{Q,gen})$	44 ($r_{Q2}=14$, $r_{Q5}=30$)	$14_{(1)}(r_{Q5}=14)$; $9_{(2)}(r_{Q4}=4, r_{Q5}=5)$; $10_{(3)}(r_{Q4}=6, r_{Q5}=4)$; $23_{(4)}(r_{Q4}=2, r_{Q5}=21)$; $1_{(7)}(r_{Q4}=1)$; $26_{(8))}(r_{Q4}=26)$; $111_{(9)}(r_{Q2}=107, r_{Q4}=4)$	$32_{(1)}(r_{Q2}=16, r_{Q5}=16)$; $10_{(2)}(r_{Q5}=10)$; $8_{(3)}(r_{Q5}=8)$; $9_{(4)}(r_{Q5}=9)$; $18_{(5)}(r_{Q4}=8, r_{Q5}=10)$; $5_{(7)}(r_{Q2}=3, r_{Q4}=2)$; $46_{(8)}(r_{Q2}=44, r_{Q4}=2)$;

QTEP (DTEP produces 10.9M$ and 14.8M$ savings regarding the STEP and QTEP approaches). The final plan obtained using DTEP for the Garver 6-buses system is illustrated in Fig. 4, where both the transmission circuits and shunt compensation added to the system for each year are shown.

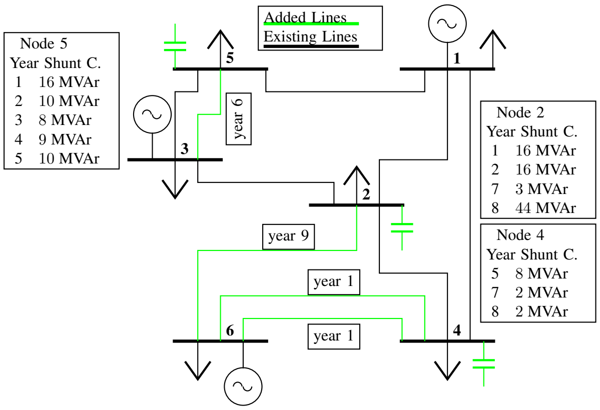

Fig. 4. Final plan from DTEP for Garver's 6-bus system (scenario B1).

3) Performance comparisons of metaheuristics: The performance of IDE-PBILc, DE-PBILc, and LPSO metaheuristics for scenario B is shown in Table VII. All metaheuristics proved to be robust when applied to STEP and QTEP obtaining a success rate of 100%. As for DTEP, even when IDE-PBILc loses some of its robustness, it is able to find the least cost solution in 70% of the trials. Different is the case of DE-PBILc and LPSO which can not find the lowest cost solution found by IDE-PBILc. Evidently, DTEP requires more computing time to be solved than STEP and QTEP due to the larger number of decision variables. Finally, IDE-PBILc required the lowest computing time for all TNEP problems and this is achieved thanks to identifying and not evaluating repeated individuals.

C. IEEE 24-bus system

1) Comparison of scenarios C and D for DTEP: Table VIII show the results of the DTEP for the IEEE 24-bus system. Similar to the results on the previous test system, scenarios D (using shunt compensation) present the possibility of avoiding to build additional transmission circuits by placing shunt compensation in certain buses. For this system, the most expensive scenarios are those which considers contingency conditions (scenarios C2 and D2).

Table VII

IDE-PBILc, DE-PBILc, and LPSO performance for the Garver 6-bus system (STEP, QTEP and DTEP)

	Scenario B1								
Metaheuristic	**IDE-PBILc**			**DE-PBILc**			**LPSO**		
TEP	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**
Success Rate (%)	100	100	70	100	100	20	100	100	20
Average Iter.	17	45	216	16	73	2	14	63	579
Lowest Cost (M$)	110.4	114.3	99.4	110.4	114.3	1598	110.4	114.3	271.9
Time (h)	3.2 (min)	0.6	5.8	12 (min)	2.2	2.1	10 (min)	1.8	6.2

Table VIII

DTEP results for IEEE 24-bus systems (scenarios C1, C2, D1, and D2).

Scenario	**C1**	**D1**	**C2**	**D2**
TEP	**DTEP**	**DTEP**	**DTEP**	**DTEP**
Added Lines$_{(year)}$	$l_{6-10}=1_{(4)}$ $l_{7-8}=1_{(8)}$ $l_{14-16}=1_{9}$	$l_{6-10}=1_{(5)}$ $l_{7-8}=1_{(9)}$ $l_{7-8}=1_{(10)}$	$l_{6-10}=1_{(1)}$ $l_{6-10}=1_{(4)}$ $l_{7-8}=1_{(7)}$ $l_{14-16}=1_{(9)}$	$l_{6-10}=1_{(1)}$ $l_{6-10}=1_{(4)}$ $l_{14-16}=1_{(7)}$
Lines Added	3	3	4	3
Total Cost (M$)	65.83	45.5	82.3	76.7
Lines Cost (M$)	65.83	36.6	82.3	72.9
Total Shunt Comp. Cost (M$)	-	8.8	-	3.86
Total Capacitive Comp. (MVAr)	-	1174	-	545
Cap. Comp. (MVAr) $r_{year}(r_{Q,gen})$	-	$196_{(4)}(r_{Q10}=131$, $r_{Q24}=65)$; $490_{(8)}(r_{Q8}=80$, $r_{Q12}=410)$; $488_{(10)}(r_{Q5}=29$, $r_{Q10}=7$, $r_{Q11}=22, r_{Q15}=216$, $r_{Q24}=214)$	-	$22_{(8)}(r_{Q8}=22)$; $73_{(9)}(r_{Q8}=44, r_{Q12}=29)$; $450_{(10)}(r_{Q5}=37$, $r_{Q8}=137$, $r_{Q10}=127$, $r_{Q11}=17$, $r_{Q12}=128$, $r_{Q24}=4)$
Saving in invest. by shunt comp. (M$)		20.3		5.6

2) Comparison among STEP, QTEP and DTEP for IEEE 24-bus system (Scenario D1): Table IX shows the DTEP and QTEP approaches present the same final plan with the best economic benefit if compared to STEP.

3) Performance comparisons of metaheuristics for IEEE 24-bus system: Table X shows the performance of IDE-PBILc, DE-PBILc, and LPSO for scenario D1, where only IDE-PBILc proved to be very robust for all TNEP approaches with a success rate of 100%. On the other hand, DE-PBILc and LPSO proved full robustness only for STEP and QTEP, while for DTEP those metaheuristics lose their robustness, and even

Table IX

TNEP results for IEEE 24-bus systems considering only circuits cost and shunt compensation (scenario D1).

Scenario	**D1**		
TNEP	**STEP**	**QTEP**	**DTEP**
Added Lines$_{(year)}$	$l_{6-10}=1$ $l_{7-8}=2$	$l_{6-10}=1_{(5)}$ $l_{7-8}=1_{(9)}$ $l_{7-8}=1_{(10)}$	$l_{6-10}=1_{(5)}$ $l_{7-8}=1_{(9)}$ $l_{7-8}=1_{(10)}$
Lines Added	3	3	3
Total Cost (M$)	59.8	45.5	45.5
Lines Cost (M$)	48	36.6	36.6
Total Shunt Comp. Cost (M$)	11.8	8.8	8.8
Total Capacitive Comp. (MVAr)	1189	1174	1174
Cap. Comp. (MVAr) $r_{year}(r_{Q,gen})$	$1189(r_{Q5}=37$, $r_{Q10}=140$, $r_{Q11}=39$, $r_{Q12}=686$, $r_{Q24}=287)$	$196_{(4)}(r_{Q10}=131$, $r_{Q24}=65)$; $490_{(8)}(r_{Q8}=80$, $r_{Q12}=410)$; $488_{(10)}(r_{Q5}=29$, $r_{Q10}=7$, $r_{Q11}=22, r_{Q15}=216$, $r_{Q24}=214)$	$196_{(4)}(r_{Q10}=131$, $r_{Q24}=65)$; $490_{(8)}(r_{Q8}=80$, $r_{Q12}=410)$; $488_{(10)}(r_{Q5}=29$, $r_{Q10}=7$, $r_{Q11}=22, r_{Q15}=216$, $r_{Q24}=214)$

Table X

IDE-PBILc, DE-PBILc, and LPSO performance for the IEEE 24-bus system (STEP, QTEP and DTEP).

Scenario D1									
Metaheuristic	**IDE-PBILc**			**DE-PBILc**			**LPSO**		
TEP	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**
Success Rate (%)	100	100	100	100	100	20	100	100	20
Average Iter.	18	104	347	57	388	2	34	208	740
Lowest Cost (M$)	59.88	45.5	45.5	59.88	45.5	5569	59.88	45.5	3262
Time (h)	5.7 (min)	0.9	0.8	20.6 (min)	3.9	3	19.6 (min)	3.3	10

they can not find the least cost solution found by IDE-PBILc. IDE-PBILc required the lowest computing time for all TNEP problems due to the implemented improvements.

D. IEEE 118-bus system

The data presented in [2] has been modified since the original base case does not require the addition of transmission lines. Therefore, the line ratings have been reduced to create line congestion in the initial network. Additionally, the line cost of 338 kV lines was increased since in [2] its value is too low compared to those of lower voltage levels. The analyzed scenarios for this test system comprise controllable and non-controllable generation. Non-controllable generation can model renewable generation (i.e. wind, solar generation), which for this system, 18% of renewable energy penetration was assumed; 9% of photovoltaic generation (nodes 4, 10, 18, 24, and 36) and 9% of wind generation (nodes 46, 72, 82, 92, and 116). With the changes made to this system, the main data for this system can be seen in Table III.

1) Comparison of scenarios E and F for DTEP: Table XI show the results of the DTEP for the IEEE 118-bus system. Similar to the results on the previous test systems, scenarios E (using shunt compensation) present the possibility of avoiding to build additional transmission circuits by co-optimizing transmission circuits and reactive power compensation. For this system, the most expensive scenarios are those which considers contingency conditions (scenarios E2 and F2).

2) Comparison among STEP, QTEP and DTEP for IEEE 118-bus system (Scenario F1): Table XII shows the DTEP approach presents the same final plan with the best economic benefit if compared to STEP and QTEP (DTEP produces 26.9M$ and 0.6M$ savings regarding the STEP and QTEP approaches). This result offers the possibility to know the best years for investments.

3) Performance comparisons of metaheuristics for IEEE 118-bus system: Table XIII shows the performance of metaheuristics IDE-PBILc, DE-PBILc, and LPSO for scenario F1 where only IDE-PBILc metaheuristic proved to be very robust for the STEP and QTEP prblems, while its robustness is reduced for DTEP. On the other hand, metaheuristic LPSO proved a low robustness even for STEP, while for QTEP and DTEP this metaheuristic can not even find the least cost solution found by IDE-PBILc metaheuristic. Different from IDE-PBILc, DE-PBILc and LPSO have no means to escape from low quality local optima and stagnate in higher cost values. Finally, IDE-PBILc required the lowest computational time for all TNEP problems showing drastic improvements if compared to the original DE-PBILc version .

Table XI

DTEP results for IEEE 118-bus systems (scenarios E1, E2, F1 and F2).

Scenario	**E1**	**F1**	**E2**	**F2**
TEP	**DTEP**	**DTEP**	**DTEP**	**DTEP**
Added Lines$_{(year)}$	$l_{8-9} = 1_{(1)}$ $l_{8-5} = 1_{(1)}$ $l_{1-3} = 1_{(3)}$ $l_{3-5} = 2_{(3)}$ $l_{8-5} = 1_{(3)}$ $l_{9-10} = 1_{(3)}$ $l_{94-100} = 1_{(7)}$ $l_{25-27} = 1_{(8)}$ $l_{15-17} = 1_{(9)}$ $l_{49-66} = 1_{(9)}$ $l_{23-25} = 1_{(10)}$ $l_{49-66} = 1_{(10)}$ $l_{77-78} = 1_{(10)}$ $l_{17-113} = 1_{(10)}$	$l_{8-5} = 1_{(1)}$ $l_{8-9} = 1_{(3)}$ $l_{9-10} = 1_{(3)}$ $l_{17-113} = 1_{(7)}$ $l_{94-100} = 1_{(8)}$ $l_{15-17} = 1_{(9)}$ $l_{25-27} = 1_{(9)}$ $l_{23-25} = 1_{(10)}$	$l_{8-9} = 1_{(1)}$ $l_{8-5} = 1_{(1)}$ $l_{1-3} = 1_{(3)}$ $l_{3-5} = 2_{(3)}$ $l_{8-5} = 1_{(3)}$ $l_{9-10} = 1_{(3)}$ $l_{77-78} = 1_{(4)}$ $l_{26-30} = 1_{(6)}$ $l_{38-37} = 1_{(6)}$ $l_{94-100} = 1_{(10)}$	$l_{8-5} = 1_{(1)}$ $l_{8-9} = 1_{(3)}$ $l_{9-10} = 1_{(3)}$ $l_{23-25} = 1_{(7)}$ $l_{94-100} = 1_{(7)}$ $l_{42-49} = 1_{(8)}$ $l_{25-27} = 1_{(9)}$ $l_{15-17} = 1_{(10)}$ $l_{17-113} = 1_{(10)}$
Lines Added	15	8	11	9
Total Cost (M$)	229.2	162.7	299.9	201.6
Lines Cost (M$)	229.2	156.4	299.9	194.1
Total Shunt Comp. Cost (M$)	-	6.3	-	7.5
Total Capacitive Comp. (MVAr)	-	475	-	544
Cap. Comp. (MVAr) $r_{year}(r_{Q,gen})$	-	$150_{(7)}(r_{Q1} = 49, r_{Q13} = 15,$ $r_{Q21} = 8, r_{Q22} = 5, r_{Q51} = 10,$ $r_{Q86} = 63);$ $31_{(8)}(r_{Q1} = 6, r_{Q21} = 6,$ $r_{Q22} = 19);$ $294_{(10)}(r_{Q2} = 7, r_{Q3} = 14,$ $r_{Q13} = 10, r_{Q16} = 1, r_{Q20} = 24,$ $r_{Q21} = 12, r_{Q28} = 21, r_{Q33} = 1,$ $r_{Q41} = 6, r_{Q43} = 5, r_{Q44} = 34,$ $r_{Q45} = 73, r_{Q51} = 5, r_{Q78} = 46,$ $r_{Q114} = 20, r_{Q117} = 8, r_{Q118} = 7)$	-	$13_{(6)}(r_{Q1} = 13);$ $105_{(7)}(r_{Q20} = 2, r_{Q21} = 50,$ $r_{Q22} = 8, r_{Q44} = 45);$ $15_{(9)}(r_{Q51} = 6, r_{Q78} = 9),$ $411_{(10)}(r_{Q1} = 36, r_{Q2} = 8,$ $r_{Q3} = 16, r_{Q13} = 25, r_{Q16} = 2,$ $r_{Q20} = 4, r_{Q22} = 7, r_{Q28} = 21,$ $r_{Q33} = 3, r_{Q35} = 13, r_{Q37} = 17,$ $r_{Q38} = 44, r_{Q41} = 7, r_{Q43} = 1,$ $r_{Q45} = 44, r_{Q51} = 44, r_{Q78} = 40,$ $r_{Q86} = 43, r_{Q114} = 23, r_{Q117} = 8,$ $r_{Q118} = 5)$
Total Induc. Comp. (MVAr)	-	374	-	489
Ind. Comp. (MVAr) $r_{year}(r_{Q,gen})$	-	$43_{(1)}(r_{Q9} = 43);$ $31_{(2)}(r_{Q9} = 31);$ $41_{(7)}(r_{Q30} = 31, r_{Q38} = 10);$ $15_{(8)}(r_{Q30} = 6, r_{Q38} = 9);$ $244_{(10)}(r_{Q23} = 2, r_{Q30} = 72,$ $r_{Q38} = 9, r_{Q64} = 161)$	-	$44_{(1)}(r_{Q9} = 44);$ $31_{(2)}(r_{Q9} = 31);$ $75_{(6)}(r_{Q38} = 75);$ $99_{(7)}(r_{Q30} = 93, r_{Q38} = 6);$ $240_{(10)}(r_{Q23} = 2, r_{Q30} = 21,$ $r_{Q64} = 215, r_{Q95} = 2)$
Saving in invest. by shunt comp. (M$)		66.5		98.3

Table XII

TNEP results for IEEE 118-bus systems considering only circuits cost and shunt compensation (scenario F1).

Scenario	**F1**		
TEP	**STEP**	**QTEP**	**DTEP**
Added Lines$_{(year)}$	$l_{8-9} = 1$ $l_{8-5} = 1$ $l_{9-10} = 1$ $l_{15-17} = 1$ $l_{23-25} = 1$ $l_{25-27} = 1$ $l_{94-100} = 1$ $l_{17-113} = 1$	$l_{8-5} = 1_{(1)}$ $l_{8-9} = 1_{(3)}$ $l_{9-10} = 1_{(3)}$ $l_{94-100} = 1_{(8)}$ $l_{25-27} = 1_{(9)}$ $l_{15-17} = 1_{(10)}$ $l_{23-25} = 1_{(10)}$ $l_{17-113} = 1_{(10)}$	$l_{8-5} = 1_{(1)}$ $l_{8-9} = 1_{(3)}$ $l_{9-10} = 1_{(3)}$ $l_{17-113} = 1_{(7)}$ $l_{94-100} = 1_{(8)}$ $l_{15-17} = 1_{(9)}$ $l_{25-27} = 1_{(9)}$ $l_{23-25} = 1_{(10)}$
Lines Added	8	8	8
Total Cost (M$)	189.6	163.3	162.7
Lines Cost (M$)	181.6	155.7	156.4
Total Shunt Comp. Cost (M$)	8	7.7	6.3
Total Capacitive Comp. (MVAr)	452	534	475
Cap. Comp. (MVAr) $r_{year}(r_{Q,gen})$	$452(r_{Q1} = 49, r_{Q2} = 8, r_{Q3} = 17,$ $r_{Q13} = 24, r_{Q16} = 2, r_{Q20} = 22,$ $r_{Q21} = 44, r_{Q22} = 16, r_{Q28} = 22,$ $r_{Q33} = 1, r_{Q41} = 7, r_{Q43} = 5,$ $r_{Q44} = 34, r_{Q45} = 74, r_{Q78} = 49,$ $r_{Q86} = 42, r_{Q114} = 22,$ $r_{Q117} = 9, r_{Q118} = 5)$	$205_{(7)}(r_{Q1} = 55, r_{Q13} = 20,$ $r_{Q21} = 9, r_{Q22} = 33, r_{Q51} = 14,$ $r_{Q86} = 72, r_{Q117} = 2);$ $109_{(8)}(r_{Q1} = 10, r_{Q13} = 3,$ $r_{Q21} = 27, r_{Q23} = 19, r_{Q38} = 2,$ $r_{Q44} = 43, r_{Q78} = 4, r_{Q117} = 1);$ $12_{(9)}(r_{Q78} = 12);$ $208_{(10)}(r_{Q2} = 4, r_{Q3} = 8,$ $r_{Q13} = 4, r_{Q16} = 2, r_{Q20} = 18,$ $r_{Q28} = 21, r_{Q33} = 3, r_{Q35} = 1,$ $r_{Q38} = 28, r_{Q41} = 6, r_{Q43} = 2,$ $r_{Q45} = 47, r_{Q51} = 1, r_{Q78} = 28,$ $r_{Q114} = 22, r_{Q117} = 5,$ $r_{Q118} = 8)$	$150_{(7)}(r_{Q1} = 49, r_{Q13} = 15,$ $r_{Q21} = 8, r_{Q22} = 5, r_{Q51} = 10,$ $r_{Q86} = 63);$ $31_{(8)}(r_{Q1} = 6, r_{Q21} = 6,$ $r_{Q22} = 19);$ $294_{(10)}(r_{Q2} = 7, r_{Q3} = 14,$ $r_{Q13} = 10, r_{Q16} = 1, r_{Q20} = 24,$ $r_{Q21} = 12, r_{Q28} = 21, r_{Q33} = 1,$ $r_{Q41} = 6, r_{Q43} = 5, r_{Q44} = 34,$ $r_{Q45} = 73, r_{Q51} = 5, r_{Q78} = 46,$ $r_{Q114} = 20, r_{Q117} = 8, r_{Q118} = 7)$
Total Induc. Comp. (MVAr)	348	487	374
Ind. Comp. (MVAr) $r_{year}(r_{Q,gen})$	$348(r_{Q9} = 26, r_{Q23} = 3,$ $r_{Q30} = 102, r_{Q38} = 26,$ $r_{Q64} = 189, r_{Q95} = 2)$	$43_{(1)}(r_{Q9} = 43);$ $31_{(2)}(r_{Q9} = 31);$ $63_{(7)}(r_{Q30} = 18, r_{Q38} = 19,$ $r_{Q94} = 26);$ $11_{(8)}(r_{Q30} = 11);$ $58_{(9)}(r_{Q22} = 11, r_{Q30} = 1,$ $r_{Q38} = 46);$ $281_{(10)}(r_{Q22} = 4, r_{Q23} = 23,$ $r_{Q30} = 77, r_{Q64} = 175, r_{Q95} = 2)$	$43_{(1)}(r_{Q9} = 43);$ $31_{(2)}(r_{Q9} = 31);$ $41_{(7)}(r_{Q30} = 31, r_{Q38} = 10);$ $15_{(8)}(r_{Q30} = 6, r_{Q38} = 9);$ $244_{(10)}(r_{Q23} = 2, r_{Q30} = 72,$ $r_{Q38} = 9, r_{Q64} = 161)$

Table XIII

IDE-PBILc, DE-PBILc, and LPSO performance for IEEE 118-bus system (STEP, QTEP and DTEP).

Scenario F1									
Metaheuristic	**IDE-PBILc**			**DE-PBILc**			**LPSO**		
TEP	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**	**STEP**	**QTEP**	**DTEP**
Success Rate (%)	80	100	60	60	100	20	40	100	20
Average Iter.	92	252	429	256	854	250	272	795	1025
Lowest Cost (M$)	189.6	163.3	162.7	189.6	163.3	11356	189.6	165.9	9030.6
Time (h)	0.6	7.1	3.6	0.9	9.4	7.7	1.8	17.5	24

E. Discussion of the computational performance

It is often not easy to directly compare the computation times to existing approaches, as the scope of the presented studies, and the used test cases are not unified. Dehghan et. al. report a computation time of 225s for the solution of the static AC TNEP problem [44], whereas Rahmani et. al. report computation times between 569 s and 1544 s using different implementations of Benders decomposition [45]. Puvvada et. al. solve a robust AC TNEP problem with computation in approximately 2900 s for the 6-bus Garver system [46]. The results provided in this work suggest that the proposed meta-heuristic approach can be a computationally efficient alternative to classical mathematical approaches, especially for larger systems due to the greedy nature of meta-heuristics in exploring the search space. However, this needs to be analysed in a more rigorous way, using a unified set of test-systems and implementations of various mathematical approaches using different kinds of relaxations and linearizations, for which the authors are currently developing a framework.

VII. CONCLUSIONS

In this paper, a methodology for security constrained dynamic transmission expansion planning (DTEP) has been provided considering investments for reactive power management. The scenarios where reactive power expansion was considered presented important savings if compared to those without shunt compensation. Also, the results of the developed methodology have been compared to the static (STEP), and pseudo-dynamic (QTEP) approaches. When DTEP and STEP were compared, DTEP presents a better economic benefit for all scenarios, while for the DTEP and QTEP comparison, the results obtained by DTEP present at least the same or even better economic benefit. A comparison among three metaheuristic optimization techniques was carried out to demonstrate the advantages of the developed method. Even when a reduction in the robustness of metaheuristics was presented for the more complex system, the results of the DTEP formulation solved by the improved version of DE-PBILc proved to be, in general, better than using the other approaches. Additionally, the advantage of identifying repeated topologies generated by IDE-PBILc during the optimization process allows significant savings of simulation time. Non-convex AC based models can not guarantee an optimal solution has been found, however, convergence to good quality solutions is always obtained using metaheuristics techniques. That is still not possible even for state-of-art simpler static AC convex models, where convergence for different scenarios, and similar test systems as those used in this work, is still an issue. More research work is necessary in order to increase the performance of the optimization technique for transmission expansion planning problems, including the direct inclusion of uncertainty in the optimization process.

APPENDIX A
DATA FOR THE 4-BUS SYSTEM

For the 4-bus system, the data of demand and generation for the three stages can be seen in the Tables XIV and XV, respectively. Branch data is presented in Table XVI.

Table XIV

Bus data for the 4-bus system.

Bus Data							
		P_D^t			Q_D^t		
Bus number	**Bus type**	P_D^1	P_D^2	P_D^3	Q_D^1	Q_D^2	Q_D^3
1	3	44.4	47.1	50	27.5	29.2	30.99
2	1	151.2	160.3	170	93.7	99.3	105.35
3	1	177.9	188.6	200	110.3	116.9	123.94
4	2	71.1	75.4	80	44.1	46.7	49.58

Table XV

Generation data for the 4-bus system.

Generator Data												
	$\overline{Q}_G^t$			$\underline{Q}_G^t$			$\overline{P}_G^t$			$\underline{P}_G^t$		
Bus number	$\overline{Q}_G^1$	$\overline{Q}_G^2$	$\overline{Q}_G^3$	$\underline{Q}_G^1$	$\underline{Q}_G^2$	$\underline{Q}_G^3$	$\overline{P}_G^1$	$\overline{P}_G^2$	$\overline{P}_G^3$	$\underline{P}_G^1$	$\underline{P}_G^2$	$\underline{P}_G^3$
1	88.9	94.3	100	-88.9	-94.3	-100	0	0	0	0	0	0
4	88.9	94.3	100	-88.9	-94.3	-100	495.2	525	556.5	0	0	0
2 (fictitious gen.)	1000	1000	1000	-1000	-1000	-1000	1000	1000	1000	0	0	0
3 (fictitious gen.)	1000	1000	1000	-1000	-1000	-1000	1000	1000	1000	0	0	0

Table XVI

Branch data for the 4-bus system.

Branch Data (100 MW base)							
From	**to**	r_k	x_k	b_k	S^{max}	**Status**	c_{kl} **(M$)**
1	**2**	0.01008	0.0504	0.1025	150	1	40
1	**3**	0.00744	0.0372	0.0775	150	1	38
2	**4**	0.00744	0.0372	0.0775	150	1	60
3	**4**	0.01272	0.0636	0.1275	150	1	20

ACKNOWLEDGEMENTS

The work is supported by the energy transition funds project NEPTUNE organized by the FPS economy, S.M.E.s, Self-employed and Energy, and VLIR-UOS, Belgium.

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