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Article

Design, Construction and Validation of a Rubric to Evaluate Mathematical Modelling in School Education

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Abstract: This study describes the design, construction and validation of a rubric for assessing mathematical modelling processes throughout schooling (3–18 years), especially those oriented by modelling cycles. The final version of the "Rubric for Evaluating Mathematical Modelling Processes" (REMMP) consists of seven elements with their respective performance criteria or items, corresponding to the different phases of a modelling cycle. We concluded that REMMP can be used by both researchers and teachers at different educational levels from kindergarten to high school. The rubric is designed to assess group work developed by students; however, it can eventually be used individually.

Keywords: mathematical modelling; mathematical practice; modelling cycle; modelling evaluation; rubric

MSC: 97D60; 97M10

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1. Introduction

Mathematical modelling, along with the introduction of information and communication technology, is probably one of the most prominent common features in maths curricula around the world in recent decades (Kaiser, Blomhøj and Sririman [1]). The progressive incorporation of mathematical modelling in the Common Core State Standards Mathematics (CCSSM) of the United States (National Governors Association for Best Practices and Council of Chief State School Officers [2]), the importance attached to this issue by documents from the National Council of Teachers of Mathematics (NCTM [3–5]) and the varied literature produced by international organizations and communities, such as the International Commission on Mathematical Instruction (ICMI), the International Community of Teachers of Mathematical Modeling and Applications (ICTMA) or the Reunión Latinoamericana de Matemática Educativa (Latin American Meeting of Educational Mathematics) (RELME), among others, are all examples of this.

This increase in the presence of mathematical modelling, both in various internationally renowned organizations and in specialized literature, is largely due to the importance of mathematical modelling both in real-life applications and in mathematical education itself. This has resulted in the ever-increasing use of mathematical modelling in contemporary curricular documents and in the language used by teachers. Despite the existence of an increasing number of studies that allow us to investigate and learn about the mathematical modelling process (Albarracín and Gorgorió [6]; Alsina, Á., Toalongo, Trelles and Salgado [7]; Bliss and Libertini [8]; Blum and Borromeo [9]; Carreira, Amado, Lecoq [10]; Ortiz, Rico and Castro [11]; Toalongo, Alsina, Á., Trelles and Salgado [12]; Trelles, Toalongo and Alsina, Á. [13]; Trelles, Toalongo and Alsina, Á. [14]; Wess and Greefrath [15]), it is also true that the curricula in many countries do not offer clear guidelines that allow teachers to implement mathematical modelling in the classroom and outline how it should be evaluated (Trelles and Alsina, Á. [16]).

This situation is accentuated in preschool and primary education levels, as the literature has focused mainly on the educational levels of secondary education, baccalaureate and higher education. In addition, it should be noted that the work of researchers has been mainly focused on aspects of the implementation of mathematical modelling rather than its evaluation. In this regard, it is demonstrated in the literature that the production of research in evaluation is very limited. For example, after studying 700 articles related to mathematical modelling, Frejd [17] found that only 10% were related to evaluation processes.

From this perspective, the aim of this study is to design, build and validate an instrument that serves teachers both in terms of guidance and evaluation. It allows us, on the one hand, to discover how mathematical modelling develops throughout the different educational stages, and on the other hand, to assess the degree of acquisition of this skill by students. To achieve this dual purpose, we opted for the design of an instructional rubric in the sense proposed by Andrade [18].

2. Theoretical Background

In accordance with the final purpose of our study, a review of the literature on two interrelated aspects was undertaken: (1) definition of mathematical modelling and (2) the presence of mathematical modelling in the main curricular documents.

2.1. Mathematical Modelling: Conceptualization and Perspectives

Although there is no single criterion in the scientific community to define mathematical modelling, some authors provide significant contributions. For example, Alsina, C., García-Raffi, Gómez and Romero [19] state that mathematical modelling refers to the process of building a model that can be used to explain or study a real or mathematical phenomenon, which requires constant translations between reality and mathematics. Villa [20] understands mathematical modelling as an activity, the nature of which is derived from the scientific action of mathematical modelling and which becomes, rather than a tool for building new mathematical objects, a strategy that enables the understanding of a mathematical concept immersed in a "microworld" that prepares the student to develop a different attitude towards asking about and addressing the problems of the real context. In this sense, Borba and Villarreal [21] point out that mathematical modelling can help make mathematics more understandable by bringing contextualized situations to the classroom and giving meaning to the mathematics that are taught and learned.

Accordingly, Bliss and Libertini [8] and Blum and Borromeo [9] conceptualize mathematical modelling as a process that uses mathematics to represent, analyse, make predictions or provide information about real-world phenomena and perform a process of translation between this world and mathematics. This is the definition of modelling that was assumed in this study. Additionally, Blum and Borromeo [9] point out that, through modelling, students can better understand the contexts in which they operate; mathematics learning is supported, and the development of some appropriate competencies, attitudes and visions towards them is promoted. This idea, in turn, is complemented by Trigueros [22], for whom the results of research show that when the concepts of mathematics are directly learned, it is not easy to apply them to problem solving.

In addition, it is important to indicate that a variety of perspectives is represented in the international debate with respect to mathematical modelling. For example, Kaiser and Sriraman [23] propose six modelling perspectives:

(1) Realistic or applied modelling (pragmatic–utilitarian goals, i.e., solving real-world problems, understanding the real world and promotion of modelling competencies); (2) contextual modelling (subject-related and psychological goals, i.e., solving word problems); (3) educational modelling (pedagogical and subject-related goals, (a) structuring of learning processes and promotion thereof, (b) concept introduction and development); (4) sociocritical modelling (pedagogical goals such as critical understanding of the surrounding world); (5) epistemological or theoretical modelling (theory-oriented goals, i.e., promotion of theory development); and (6) cognitive modelling (research aims: (a) analysis

of cognitive processes taking place during modelling processes and understanding of these cognitive processes; psychological goals: (b) promotion of mathematical thinking processes by using models as mental images or even physical pictures or by emphasizing modelling as mental process such as abstraction or generalization). (p. 304)

Although this classification is an important contribution, Trigueros [22] states that in current studies on modelling, it is difficult to find examples that fall into a single category. For this author, even when it is possible to classify them within one of these perspectives, elements that can be considered to belong to other elements will always be present. This idea is corroborated by Blomhøj [24] and Abassian, Safi, Bush and Bostic [25], who state that the perspectives share commonalities and therefore overlap.

2.2. The Modelling Cycle

One of the consensuses in literature regarding this issue is that mathematical modelling is a non-linear and iterative process. In fact, there are several authors who propose that mathematical modelling processes develop through cycles (Carreira, Amado and Lecoq [10]; Geiger [26]; Girnat and Eichler [27]; Greefrath [28]; Kaiser [29]). Although these approaches have characteristics in common, in this study, we adopted the cycle proposed by Blum and Leiβ [30] (Figure 1).

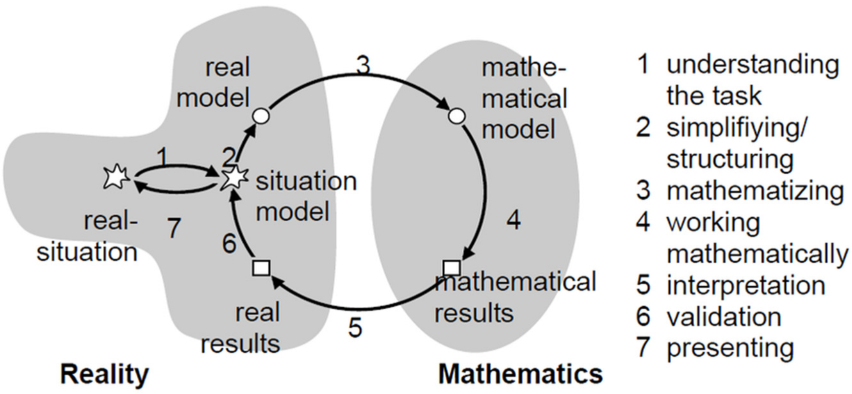


Figure 1. Mathematical modelling cycle proposed by Blum and Leiβ [30].

For Czocher [31], in this modelling cycle, the real situation occurs in the real world. Working to understand the problem produces a situation model, i.e., a conceptual model in the mind of the modeller. Simplifying/structuring refers to identifying, introducing and specifying variables and conditions. This specifies the real model (which likely has internal and external components). Through mathematization, the modeller represents the real model mathematically. The mathematical model itself is an expression, in formal mathematics, of relationships among key variables. Working mathematically or performing analysis produces mathematical results, which can then be interpreted in terms of the real model in order to obtain real results. These results are then validated by checking them against the situation model. Lastly, the student exposes or shares his model with others.

Students can go from one point to another of the cycle without having to follow an established order, and it is precisely this roundabout and iterative path that allows them to refine the desired model. As can be seen in Figure 1, in the final phase, it is important for students to share the model with their classmates, collect relevant observations and make the necessary adjustments in order to continue improving the model. In this stage, the role played by the teacher is essential for students to achieve the proposed objectives.

> "Students can observe how their teacher continues to embrace new questions and actively question and seek answers. Teachers can demonstrate curiosity and persistence. This learning together of new facts, conventions, and approaches can be one of the most fun and rewarding parts of teaching mathematical modeling."

(Consortium for Mathematics and its Applications, [COMAP] & Society for Industrial and Applied Mathematics, [SIAM], [32], p. 57).

How do we make it possible for students to arrive at a position where they can generate mathematical models? According to Colwell and Enderson [33], for them to be able to use different tools that help them solve problems based on reasoning, modelling and communication of their ideas, it is necessary for initial training to provide knowledge to future teachers so they can promote those skills. For these authors, the teacher is the key, as not only does the teacher propose exercises, but it is the teacher who must also possess the knowledge and professional competence to provide support to the students. In a preliminary study, Ortiz, Rico and Castro [11] also emphasize the importance of mathematical modelling being promoted in the initial training of teachers. This is because it is at this time when mathematical concepts and procedures are acquired that will later permit the finding of solutions to everyday problems through the development of skills that the teacher can then introduce into the classroom.

Despite the benefits of mathematical modelling for both teaching and learning, several studies demonstrate the difficulties teachers encounter with its implementation. In this sense, Aydin and Özgeldi [34] and Sáenz [35] agree that teachers in training encounter difficulties when working with mathematical models to solve real problems in the sense of establishing connections between context and mathematical knowledge for their solution. Similarly, Olande [36] concludes that when solving items from PISA 2003, the activities that generate the greatest difficulty for teachers during training are those that require reflection and connection between the contents and the context, which highlights difficulties in the levels of mathematical competence.

In summary, mathematical modelling requires knowledge, skill and time, as it represents a challenge for both practising teachers and teachers in training. For this reason, it is vital to generate strategies that favour the acquisition of professional skills during teacher training (Wess and Greefrath [15]).

2.3. Mathematical Modelling in Curricular Documents

As indicated in the Introduction, mathematical modelling is increasingly present in the study plans of various countries. However, the approaches taken in each one of these countries are very different. In this sense, Blum and Niss [37] identified six types of approaches for the incorporation of mathematical modelling in study plans. The spectrum of approaches ranges from incorporating mathematical modelling into subjects other than mathematics to not teaching mathematics as an independent subject but rather integrating it into other subjects or courses to facilitate mathematical modelling.

According to Borromeo [38], in Germany, the Netherlands, the United Kingdom and Scandinavian countries, for example, the contextual and realistic approach prevails, whereas in France, Italy, Portugal and Spain, the epistemological or theoretical approach has a greater presence. in their review of curricular documents from the US, Spain and Ecuador, Trelles and Alsina, Á. [16] state that a common element in the study plans of these countries is the gradual implementation of the modelling processes, whereby specific models and visual graphic models are used predominantly in the earlier school levels (3 to 11 years); the use of previously established models of a slightly more formal nature are used more frequently in intermediate school levels (12 to 14 years); and finally, the creation of models with their respective analysis, interpretation and judgment of the modelling process is unique in the later educational levels studied (15–18 years).

In this sense, these authors consider it important that activities generating processes of criticism and reflection be developed from the earliest ages. In addition, these authors found that in the United States, the orientations of which serve as guidance for the design of mathematics study plans in many countries, the standards issued by the NCTM [4] propose working with mathematical modelling from an early age. However, at higher levels, the standards for working with modelling are not explicit.

They also point out that the CCSSM (National Governors Association for Best Practices and Council of Chief State School Officers [2]) suggests incorporation of mathematical modelling throughout different levels, although some level of disconnection is present, since there are some domains in some educational levels that do not work with mathematical modelling. In the case of Spain, the authors found that although in primary education, considerable importance is attached to problem solving, it is unfortunate that mathematical modelling is not explicitly incorporated into the official study plan. However, in the Ecuadorian study plan, it does appear, although disconnected between the different levels. This situation is considerably improved in Secondary and Baccalaureate Education in both countries, where the importance of mathematical modelling throughout the different study blocks can be seen. In general terms, there is gradual progress being made with modelling in study plans but with some problems of disconnection in the early stages that can cause teachers to not really know what to do in the classroom.

In summary, several authors have contributed to the conceptualization of mathematical modelling, which is commonly defined as a process that requires a constant translation—in both directions—between mathematics and reality. In addition, although curriculum documents in several countries have begun to incorporate mathematical modelling, guidelines for teachers to implement it in the classroom with its corresponding assessment are still scarce. In this sense, a rubric that initially helps teachers to identify the processes developed by students can help to fill this gap, bearing in mind that it will later be necessary to specify levels of acquisition of these processes.

3. Design, Construction and Validation of an Instrument for the Evaluation of Activities Involving Mathematical Modelling Processes

With the aim of providing teachers with resources to implement and analyse mathematical modelling processes in the classrooms, the "Rubric to Evaluate Mathematical Modelling Processes" (REMMP) was produced. Considering that the curricular documents establish the use of mathematical modelling at different educational levels, although with some disconnection in the early stages, as indicated above, the REMMP was designed to be used from the initial levels of education (3–5 years) up to the most advanced levels of preuniversity education (15–18 years). The rubric covers this wide age range so that those using it can easily relate the main features of a modelling cycle either at an earlier or later level of education. It is designed to serve the dual purpose indicated in the Introduction: both to make known how this process develops during the different educational stages and to evaluate the level of acquisition (Andrade [18]). Regarding evaluation, it should be noted that the REMMP is primarily designed to evaluate students' group work, as one of the main characteristics of the mathematical modelling process is collaborative work. However, the possibility that it can be used individually with students is not excluded, although the latter scenario is less frequent.

3.1. Phase 1: Bibliographic Review and Analysis of Instruments That Allow the Evaluation of Mathematical Modelling

In this phase, a review of English and Spanish literature was carried out in the main databases, such as Web of Science, Scopus and Dialnet. The search criteria were that the documents (a) were published between 2005 and 2022; (b) were articles, books or book chapters; and (c) the following keywords appeared in the title or abstract: assessment, mathematical and modelling. Papers related to mathematics education were selected. Among this selection, the documents that were most relevant to our study were those cited in this section.

The literature review conducted by Frejd [17] regarding the evaluation of mathematical modelling identified the fact that written tests, projects, practical tests, portfolios and contests are the main proposals for evaluating mathematical modelling. Some of these evaluation methods, such as written tests, are based on a more atomistic vision, focusing more on the product than on the process. On the other hand, projects tend to approach evaluations from a more holistic view.

In the case of written tests, Haines and Crouch [39] state that these elements do not address the full range of modelling skills, as they still do not cover mathematical work, the refinement of a model and the presentation of reports.

Some important guidelines for evaluating mathematical modelling processes were also found in literature, specifically the Guidelines for Assessment an Instruction in Mathematical Modelling Education (COMAP & SIAM [32]). This document contains examples of assessment tools, including checklists and rubrics, that teachers might consider using when teaching with modelling.

There are also previous studies in the literature on the assessment of mathematical modelling (Leong [40] and Tekin-Dede and Bukova-Güzel [41]), which, while providing significant data, do not consider students' developmental stages.

In addition, Turner, Roth McDuffie, Benneth, Aguirre, Chen, Foote and Smith [42] and Turner, Chen, Roth McDuffie, Smith, Aguirre, Foote and Benneth [43] make interesting contributions to the evaluation of the mathematical modelling process; however, this proposal is limited to grades 3 to 5 of elementary education.

For this reason, the study of the different proposals enabled us to corroborate that mathematical modelling is a complex process that can be approached from different perspectives and that the modelling cycle is an instrument that simplifies the staging of mathematical modelling processes with their corresponding evaluation.

3.2. Phase 2: Construction of the Initial Version

Based on our literature review, an instrument was designed in the form of a rubric directed towards each of the educational stages. In this rubric, seven elements were proposed corresponding to the different phases of the modelling cycle proposed by Blum and Leiβ [30] and described in Section 2.2. In the terms described by Sanmartí and Mas [44], these elements are the essential components of the knowledge that is intended to be analysed. In our case, these are the mathematical modelling processes, which we generically call the "Elements of the Rubric."

Once the seven elements of the rubric were established, the order of the performance criteria was identified and planned. These include the ideas that must be considered to successfully achieve the knowledge corresponding to each element of the rubric. These are generically called "items". Osterlind [45] defines items as units of measurement composed of a stimulus and a form of response, which provides information on the element to be analysed. Millman and Greene [46] add that in addition to covering the meaning of global reference of the element to be evaluated, they should ensure satisfactory validity.

In previous work, each dimension of a rubric generally proposes a single indicator, with different levels. For example, in Level 3 of the mathematization phase, Tekin-Dede and Bukova-Güzel [41] emphasize, "Constructing correct mathematical model(s) based on partially acceptable assumptions" (p. 54); in the mathematical operations process, Leong [40] indicates, "Analyzes relationships between variables" (p. 64); and COMAP & SIAM [32], in the ideal level of the solution, i.e., results are accessible to the audience dimension, indicate, "Clearly presents a solution that is consistent with the original problem statement. If appropriate, a useful visual aid/graphics in included" (p. 219). The purpose of some of these rubrics is to evaluate written reports of mathematical modelling activities, so it is relevant to construct a rubric that comprehensively evaluates the whole modelling process carried out by the students. Additionally, it is an added value to consider different indicators for each dimension, especially if the aim is to assess modelling at different educational levels.

3.3. Phase 3: Validation

The initial version underwent a content validation process through an expert judgment process. To this end, a letter of invitation was sent by e-mail to twelve experts—four from the United States and eight from Spain—taking into account that the main criterion for their selection was that they had research experience in mathematical modelling in education, either at one or several school levels. The invitation letter described the context, description and purpose of the study. The definition of modelling assumed in the study was also explicitly stated, and reviewers were requested to contribute to the present work if they fully agreed with this perspective. In addition, a guide was provided to assess the relevance of each item, and a section was included in which experts could make comments and/or suggestions on each item. Eight of the invited experts responded; of these, three were from the United States, two of whom had experience in high school and university and one in primary education; of the five reviewers from Spain, two had experience in early childhood education, two had experience in primary education and one in high school and university. The obtained results were analysed using the content validity ratio (*CVR*) initially proposed by Lawshe [47] and later modified by Tristán-López [48] (*CVR'*), according to which for an item to be validated, it must have an index greater than or equal to 0.58 after applying the following formula:

$$CVR' = \frac{n_e}{N} \tag{1}$$

where n_e = Number of experts who agree on the essential category; and N = total number of experts.

In addition, the validity index of the entire instrument (*CVI*) was obtained using the following formula:

$$CVI = \frac{\sum_{i=1}^{M} CVRi}{M} \tag{2}$$

where *CVRi* = content validity ratio of the acceptable items according to Lawshe's criteria; and *M* = total of acceptable items of the instrument.

This *CVR* index is presented in Appendix A.

3.4. Phase 4: Adjustments and Construction of the Final Version of the Rubric

In order to analyse the answers given by the experts, they were codified as Reviewer 1 (R1), Reviewer 2 (R2) and Reviewer 8 (R8). From the results obtained from the external validation process through expert judgment and subsequent internal validation through the CVR' index, the items from the initial version of the rubric were incorporated, deleted or reformulated.

First, items 1.1.c., 1.1.d., 2.1.a., 2.1.b., 2.2.b., 2.1.c., 2.2.c., 2.2.d., 3.1.a., 3.2.a., 3.1.b. and 4.1.a. were eliminated, as they did not reach the minimum value of the CVR' index needed to be classified as acceptable. In addition, as indicated, although the other items reached the necessary values, many of them were modified after taking into account the observations and suggestions made by experts on issues such as formulation. Below are some summarised comments and evaluations made by the eight experts who participated in the process.

3.4.1. Element 1: Understanding

Most evaluators suggested improving the formulation of certain items, as well as complementing others. Table 1 shows the comments received on item 1 of the REMMP.

In addition, R5 suggests incorporating an item that refers to the student being aware of the type of solution being sought, i.e., a number, a range of values, a set of values, a graph, a formula, etc.

3.4.2. Element 2: Structuring

As can be seen in Table 2, most of the reviewers' comments make reference to the reformulation of the items or their elimination, as they are aspects that do not belong to the stage.

Table 1. Comments on "Understanding".

Item	Reviewers' Comments
1.1.a. Relates the content of the problem with his or her knowledge of the environment.	R3 suggests unifying the language throughout the whole instrument. R2 considers that at no time does it stop being important for the student to use their previous knowledge; with age, that knowledge will be gradually more enriching. R7 expresses similar opinions. In addition, at the 6–12-year level, R5 proposes the reformulation of the item in terms of the student's ability to explain the problem to classmates and the teacher, showing how he/she relates the content of the problem with his/her previous knowledge.
1.2.a. Poses questions about the problem.	R3 suggests that this item should be considered at all levels, with modifications at some levels in terms of the ability to reformulate the problem.
1.1.c./1.1.d. Understands the statement of the problem to be solved.	Most reviewers agree that the item is too general for both the 12–16-year level and for the 16–18-year level and that it could be replaced by more specifics.

Table 2. Comments on "Structuring".

Item	Reviewers' Comments
2.2.b./2.1.c./2.1.d. Organizes ideas that contribute to the solution of the problem.	R3 suggests restructuring this item, emphasizing the identification of the relevant variables and the ability to relate them.
2.1.b./2.2.c./2.2.d. Proposes solution strategies.	R2, R3, R4, R5 and R8 consider that this item is not relevant because there is no relationship with the structuring phase. Specifically, R5 indicates that in this phase, the solution of the problem is not yet entered into. However, the reformulation in the real context is entered into and will be used to make the transition to the mathematical world.

Finally, R5 suggests incorporating an item related to identifying the data that are known, those that can be known and the unknown in the problem.

3.4.3. Element 3: Mathematization

In relation to the items of the third element of the rubric (Table 3), most experts comment on the formulation and relevance, that is, whether or not the items in question belong to the element. Furthermore, some experts also make interesting contributions about the location of the items in a certain educational stage.

Table 3. Comments on "Mathematization".

Item	Reviewers' Comments
3.1.c./3.1.d. Correctly identifies the variables present in the problem.	R3 and R8 agree that this item belongs to the structuring phase, as in the mathematization phase, these variables are expressed with mathematical objects according to age. Complementing this item, R6 indicates that the word "correctly" should be omitted because when referring to modelling, all ideas are valid, as they allow the student to interpret the situation through various possible solutions.
3.3.d. Formulates hypotheses and conjectures related to the problem.	R7 recommends including this item in the previous educational stage (12–16 years). R5 states that being present in the mathematization phase, it should be specified that the hypotheses and conjectures must be related to mathematical objects. R1 suggests changing the word "problem" to "situation" because the initial verbal language starts from a "real situation" with "colloquial" language.
3.2.b. Uses mathematical knowledge.	R5 suggests that the item be reformulated in the sense of the student's ability to introduce the mathematical objects as a replacement for the real elements. R4 corroborates R5, indicating that in this phase, the focus is the translation from the real world to the mathematical world.

3.4.4. Element 4: Mathematical Work

The contributions of the experts (Table 4) refer mainly to the elimination of items and/or the incorporation of items not contemplated in the initial version of the rubric.

Table 4. Comments on "Mathematical work".

Item	Reviewers' Comments
4.1.a. Interacts with classmates and teacher to discuss and validate possible solutions.	Reviewers R3, R5, R7 and R8 consider that the item is not relevant. For example, R7 states that this item overlaps with the presentation phase.

R5 suggests incorporating an item that considers the type of strategies used to solve the problem. In addition, the comments of R5 and R7 coincide with respect to the incorporation of an item that refers to obtaining a mathematical model.

3.4.5. Element 5: Interpretation

As can be seen in Table 5, the comments received refer to the reformulation of some of the items of the "interpretation" element or to the incorporation of other items.

Table 5. Comments on "Interpretation".

Item	Reviewers' Comments
5.1.a. Compares the solution with what happens in the immediate environment.	R8 suggests improving the item, indicating that the interpretation has to do with relating the results of mathematical work, that is, mathematical objects, with the reality of the context. R3 suggests similar criteria.
5.1.b./5.1.c./5.1.d. Reflects on the coherence of the mathematical results obtained.	R5 recommends improving the item, indicating that in this phase, the coherence of the mathematical solutions in the real context must be checked.

Finally, R5 suggests adding an item referring to identifying the limitations or restrictions of the mathematical solution in the real context.

3.4.6. Element 6: Validation

Regarding the items of this element, the comments received (Table 6) focus on the improvement of the manner of writing and/or the inclusion of aspects related to the validation phase of the modelling cycle not considered in the initial version of the rubric.

Table 6. Comments on "Validation".

Item	Reviewers' Comments
6.1.a. Checks the validity of the results obtained.	R3 states that the item is too general, so improvement is recommended. R5 suggests that this item should be formulated in terms of the validation of the constructed model, considering whether the initial situation is resolved completely or partially.
6.1.b./6.1.c./6.1.d. Contrasts the mathematical results with the real situation.	R5 indicates that the item can be improved and that the validation of the constructed model should be considered.

R5 recommends adding an item that considers whether the model is always valid or requires changes for it to be useful in new situations.

3.4.7. Element 7: Presentation

Table 7 shows the comments received with respect to this item.

Table 7. Comments on "Presentation".

Item	Reviewers' Comments
7.1.a. Communicates the results obtained using language in accordance with age.	R5 recommends improving this item, taking into account that in this phase, the scope of the obtained model must be explained, in addition to suggesting that the item can be unified at all levels, as also indicated by R3.

R7 recommends incorporating an item that considers communicating the developed modelling process. Additionally, V5 emphasizes the importance of justifying the decisions made in each of the phases of the process.

R5 suggests considering an item that takes into account whether or not technology was used in any phase of the process.

Finally, R1 and R5 agree that there should be an item that involves communicating the decisions that did not lead to any solution with corresponding reflections.

The final version of the REMMP developed after this review process is presented in Table 8. It is important to indicate that based on the observations made by the experts, in addition to eliminating, incorporating or improving many of the initially proposed items, it was decided to unify the two educational levels (12–16 years) and (16–18 years) due to the almost total similarity of the content of the items.

Table 8. Final version of the REMMP.

Phases	Preschool Education	Primary Education	Middle/High School
1. UNDERSTANDING	1.1.a. The content of the problem is related to previous knowledge.	1.1.b. Explains the problem to classmates and the teacher, showing how it relates the content using previous knowledge.	1.1.c. Explains the main characteristics of the problem to classmates and the teacher, relating it to their previous knowledge.
	1.2.ab. Poses questions about the problem.		1.2.c. Able to reformulate the problem.
	1.3.a. States the type of solution that the problem would generate, for example, a pattern, a number, a graph, etc.	1.3.b. States the type of solution that the problem would generate, for example, a number, a range of values, a set of values, a graph, a formula, a table, etc.	1.3.c. States the type of solution that the problem would generate, for example, a number, a range of values, a set of values, a graph, a formula, a table, the design of an object, etc.
	1.4.abc. Represents the main characteristics of the problem through drawings.		
		1.5.b. Expresses what the solution of the problem would bring to the environment.	1.5.c Reflects on the extent to which the solution of the problem would influence the environment in which it is developed.
2. STRUCTURATION	2.1.a. Identifies the main elements of the problem.	2.1.bc. Identifies the data that are known, can be known and are unknown in the problem.	
	2.2.abc. Proposes ideas and/or assumptions that contribute to the simplification of the problem.		
			2.3.c. Identifies the variables present in the problem and is able to search for relationships between them.
3. MATHEMATIZATION	3.1.abc. Replaces the real elements with mathematical objects.		
	3.2.a. Explains the use of mathematical objects.	3.2.bc. Justifies the use of mathematical objects based on the characteristics of the problem.	
		3.3.bc. Identifies all the mathematical parameters present in the problem and the relationships between them.	
			3.4.c. Formulates hypotheses and/or conjectures related to the mathematical objects of the problem.

Table 8. *Cont.*

Phases	Preschool Education	Primary Education	Middle/High School
4. MATHEMATICAL WORK	4.1.abc. Uses various strategies according to age that allow for the proposition of solutions to the problem.		
	4.2.a. Uses mathematical objects in accordance with age to solve the problem.	4.2.bc. Uses mathematical objects and operates them to solve the problem.	
	4.3.abc. Obtains an initial mathematical model as a result of previous work.		
5. INTERPRETATION	5.1.a. Compares the solution with the initial problem.	5.1.bc. Checks the coherence of the mathematical solution applied to the initial real context.	
	5.2.a. Argues the validity of the results obtained.	5.2.bc. Identifies the possible limitations or restrictions of the mathematical solution in the initial real context.	
6. VALIDATION	6.1.abc. Justifies the proposed model through valid arguments.		
	6.2.abc. Assesses whether the obtained model provides a partial or total solution to the initial problem.		
		6.3.bc. Identifies whether the model is always valid or whether changes are required to make it generalizable to new situations.	
			6.4.c. Generalizes the results, demonstrating that the model can be applied to new situations.
7. PRESENTATION	7.1.abc. Explains the reasons for the decisions made throughout each phase of the process.		
	7.2.abc. Explains the obtained model as applied in the situation of the real context, its scope and limitations using age-appropriate language.		
	7.3.abc Uses different types of examples, representations, diagrams, drawings, graphs, tables of values, symbolic language, etc.		
	7.4.abc. In the case of use of technology in one or several phases of the process, clearly states at what time, how and for what it was used.		
	7.5.abc. Listens to observations and/or suggestions raised by classmates and/or the teacher.		
	7.6.abc. Responds to the observations and/or suggestions of classmates and the teacher, using language according to their age.		
		7.7.bc. If in the process, paths were used that did not lead to any solution, reflects on them and socializes their main aspects.	
		7.8.bc. Critically analyses the presentations made by classmates.	

4. Applicability of the Rubric

In the interest of brevity, we do not present examples for each of the levels of the rubric here, nor do we describe the complete development of the activities. However, in the two selected examples (Early Childhood Education and Secondary Education) we present an extract from the analysis to show the applicability of the rubric. It should be noted that due to the complex nature of modelling processes, it is possible that students' actions can sometimes be assigned to more than one indicator in the rubric.

Example 1:

Level: *Early Childhood Education (4–5 years)*

Context and statement of the Modelling Activity: *As a result of the storm in Galicia (Spain), the newspapers and news programmes are full of news about the cold, the snowfall and the drop in temperatures, which aroused a lot of interest among the children. Based on this situation, the children were asked: "Do you know how the temperature is measured? Some of the children answered with a thermometer. Next, the question "what is a thermometer?" was asked, and some children's answers were that it is an instrument with several numbers on it.*

Therefore, the following question was posed as a guide for the whole modelling activity: how are the numbers located on a thermometer and how are they interpreted?

The activity was spread over six sessions and was audio- and video-recorded. The aim was for the children to answer the guiding question and to graphically represent a thermometer, in addition to giving meaning to the numbers on it. An extract from the analysis of the activity related to the representation of the thermometer, is presented in Table 9.

Table 9. Extract from the analysis of the modelling activity in early childhood education with the REPMM instrument.

Participants	Transcriptions	Indicator	Phase of the Cycle
Teacher:	100 above, and 100° how is it?		
Boy I:	Hot	1.1.a. Relates the content of the problem situation to their prior knowledge.	Understanding
Boy Ra:	And 0 down cold.		
Teacher:	Great, will you register it?		
Boy I:	It goes out to the White panel and registers 100 degrees hot.	4.2.a. Uses mathematical objects in accordance with age to solve the problem.	Mathematical work
Boy Ra:	It goes out and picks up 0 degrees cold.		
Teacher:	Do you know more temperatures?		
Boy S:	80.	4.2.a. Uses mathematical objects in accordance with age to solve the problem.	Mathematical work
Teacher:	What is it like?		
Boy S:	Hot. (Goes out to the panel, sets the 80 and collects the data on the thermometer.	4.2.a. Uses mathematical objects in accordance with age to solve the problem.	Mathematical work
Girl A.F.	And 1 es cold.		
Girl G:	2.		
Girl Y:	3.		
Girl A.R:	4.		
Teacher:	How are 1,2,3,4?		
Boys:	Cold.	1.1.a. Relates the content of the problem situation to their prior knowledge.	Understanding
Teacher:	How nice how many we have! Any others?		
Boy S:	75, 75 is hot.	3.1.a. Replaces the real elements with mathematical objects.	Mathematization
Teacher:	And do you know where to put 75 on the thermometer?		
Boy S:	Yes, between 70 and 80, in the middle.	4.2.a. Uses mathematical objects in accordance with age to solve the problem.	Mathematical work
Boy Ru:	70 is also hot.	1.1.a. Relates the content of the problem situation to their prior knowledge.	Understanding
Teacher:	And where does hot start?		
Boy Ru:	We wait to feel it in the glasses.	2.2.abc. Proposes ideas and/or assumptions that contribute to the simplification of the problem.	Structuring

Example 2:
Level: *Secondary Education*
Context and statement of the Modelling Activity: *The development of a Fermi realistic problem-solving activity to introduce mathematical modelling.*

The statement of the activity was the following (Figure 2):

> The Empire State Building is one of the most visited places by tourists in New York-USA. This skyscraper has an information desk on the ground floor, the two most frequently asked questions to the staff at the desk are: How long does the tourist lift take to reach the observatory on the top floor, and how long does it take if you decide to walk up the stairs?
>
> Your task is to write a letter answering these questions, including the assumptions on which you base your reasoning and the procedures used. In addition, you should indicate whether your procedure would be applicable to answer these questions for other buildings.

Figure 2. Adapted from Ärlebäck [49].

The team of students selected to exemplify the use of the rubric was a pair consisting of a boy, "D" (13 years old), and a girl, "T" (14 years old). "D" had some experience in modelling activities, whereas “T” was doing this activity for the first time. There was no time limit for the activity.

The development of the activity was audio- and video-recorded, then transcribed into a text file for analysis. The activity was completed in 57 minutes.

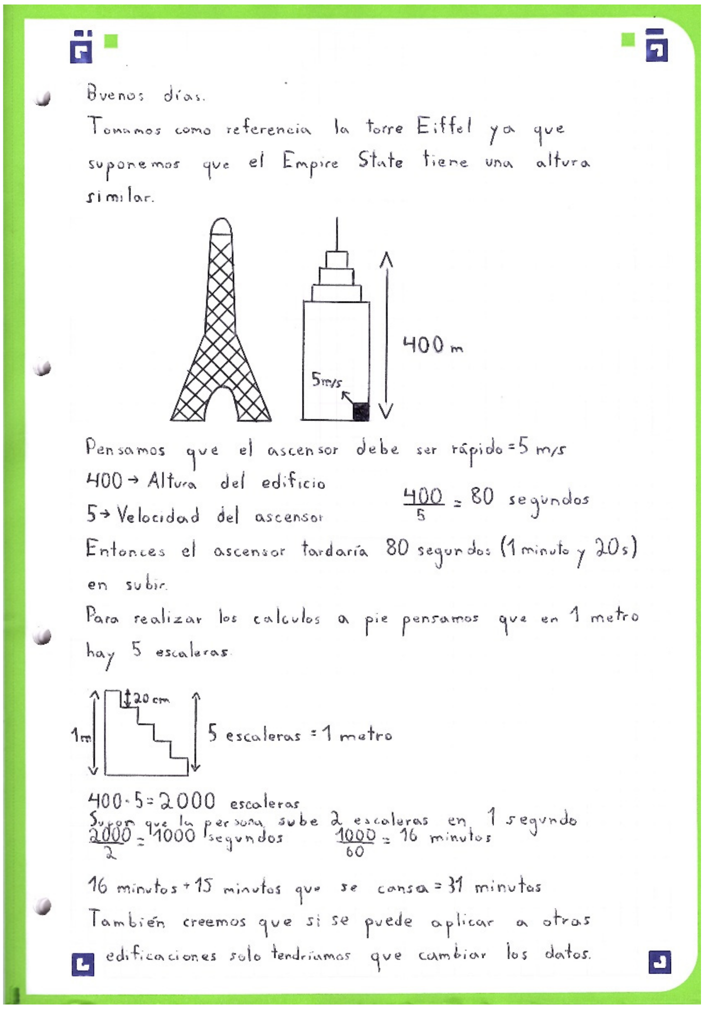
At the beginning, the students focus their discussion on establishing a plan that will allow them to arrive at the answers to the questions. The boy states that if they have the height of the building and the speed of the lift, they could determine the time needed to go up, which would allow them to answer the first question. They then use their prior knowledge to make assumptions, for example, comparing the height of the Empire State Building with that of another building, such as the Eiffel Tower, and then concentrate on executing their plan to complete the activity.

Table 10 presents an extract from the analysis of the activity related to the determination of the time it takes to go up the lift.

Table 10. Extract from the analysis of the modelling activity in compulsory secondary education with the REPMM instrument.

Student	Transcriptions	Indicator	Phase of the Cycle
D:	Let’s see T, let’s say it’s about 400 m, so, now, we have to see how fast.	2.3.c. Identifies the variables present in the problem and is able to search for relationships between them.	Structuring
T:	Yeah, uh. You see there’s always like a, a part like a button for when you’re in danger, so you go down fast.		
D:	Yeah, but I don’t think that’s for when you have to go up [pauses] I think the lift has to be kind of fast, because being such a tall building I doubt very much that it has a slow lift because otherwise it would take a long time to go up.	1.1.c. Explains the main characteristics of the problem to classmates and the teacher, relating it to their previous knowledge.	Understanding
T:	Right.		
D:	And you, how fast do you think the lift is going?	2.1.c. Identifies the data that are known, which can be known and which are unknown in the problem.	Structuring
T:	100 for every one second, I guess, no, no, it would be more or less like 50.	3.4.c. Formulates hypotheses and/or conjectures related to the mathematical objects of the problem.	Mathematization
D:	50? No, I don’t think 50.	5.1.bc. Checks the coherence of the mathematical solution applied to the initial real context.	Interpretation
T:	Yeah, I think so, because it’s going up and it’s different.	1.1.c. Explains the main characteristics of the problem to classmates and the teacher, relating it to their previous knowledge.	Understanding
D:	But you think that it would go so fast that in 10 s from the bottom, it would already be at the top. Not even 10 s.	5.2.bc. Identifies the possible limitations or restrictions of the mathematical solution in the initial real context.	Interpretation
T:	No, it would be every 5 then, 5 m.	3.4.c. Formulates hypotheses and/or conjectures related to the mathematical objects of the problem.	Mathematization
D:	Oh yeah, I see what you mean. Yes, yes, I think it would be like that, 5 m per second.		
T:	Yes, I think so.		
D:	So, now we would divide the height by the speed and that way we get the time the lift takes.	3.3.bc. Identifies all the mathematical parameters present in the problem and the relationships between them.	Mathematization
T:	So, it’s 400 m high and the lift would have a speed of 5 m per second.	3.1.abc. Replaces the real elements with mathematical objects.	Mathematization
D:	Yeah sure, 5 m per second [writes a bit, does a calculation], but if it goes at 5 m per second, uh, it would take 80 s that it does from, uh, the time that would be from the ground floor to the top, would be that, 80 s, so that would be 1 min and 20 s.	4.3.abc. Obtains an initial mathematical model as a result of previous work.	Mathematical work

In Figure 3, we present a chart of the selected student team with the respective phases of the modelling cycle.


Buenos días.
Tomamos como referencia la torre Eiffel ya que suponemos que el Empire State tiene una altura similar.

Pensamos que el ascensor debe ser rápido = 5 m/s
400 → Altura del edificio
5 → Velocidad del ascensor
$\frac{400}{5} = 80$ segundos
Entonces el ascensor tardaría 80 segundos (1 minuto y 20s) en subir.
Para realizar los calculos a pie pensamos que en 1 metro hay 5 escaleras.

400·5 = 2.000 escaleras
Supon que la persona sube 2 escaleras en 1 segundo
$\frac{2000}{2} = 1000$ segundos $\frac{1000}{60} = 16$ minutos
16 minutos + 15 minutos que se cansa = 31 minutos
También creemos que si se puede aplicar a otras edificaciones solo tendríamos que cambiar los datos.

Good morning,
We take the Eiffel Tower as a reference because we assume that the Empire State Building has a similar height.
We think that the lift must be fast = 5m/s.
400 → Building height
5 → Lift speed

$$\frac{400}{5} = 80 \text{ seconds}$$

Then the lift would take 80 seconds (1 minute and 20 seconds) to go up.
To do the calculations on foot, we think that there are 5 stairs in one metre.
400·5=2000 stairs
We assume that the person walks up 2 stairs in 1 second.

$$\frac{2\,000}{2} = 1\,000 \text{ seconds} \qquad \frac{1\,000}{60} = 16 \text{ minutes}$$

16 minutes + 15 minutes that you get tired = 31 minutes.
We also believe that it can be applied to other buildings, we would only have to change the data.

Figure 3. Original letter produced by the students and English translation.

A short section of the analysis is presented in Table 11 (other phases of the modelling cycle that were analysed above are also shown). Overall, the letter shows indicator 7.2.abc: "Explains the obtained model as applied in the situation of the real context, its scope and limitations using age-appropriate language".

Table 11. Extract from the analysis of the modelling phases presented in the chart with the REPMM instrument.

Fragment	Indicator	Phase of the Cycle
We take the Eiffel Tower as a reference since we assume that the Empire State Building is of similar height.	7.1.abc. Explain the reasons for the decisions made throughout each of the phases of the process.	Exposition/Presentation
Drawings representing the height of the Eiffel Tower and the Empire State Building and drawing representing the number of stairs in a metre.	7.3.abc. Uses different types of examples, representations, diagrams, drawings, graphs, tables of values, symbolic language, etc.	Exposition/Presentation
We also believe that it can be applied to other buildings, we would only have to change the data.	6.4.c. Generalizes the results by demonstrating that the model can be applied to new situations.	Validation

In summary, the use of the rubric shows how students move through the different phases of the modelling cycle. Specifically, the data in Tables 9–11 show that this first analysis of two mathematical modelling activities using the REPMM instrument can provide useful information for teachers about the progressive levels of mastery of students with respect to the creation of a model. Moreover, it can also be useful for the students themselves, especially at higher levels, to know objectively which phases of the modelling process need to be improved, using the rubric as a self- or co-assessment tool.

5. Final Considerations

In this study, the design, construction and validation of the rubric REMMP 3–18 was presented, with the aim of providing teachers and the scientific community with a useful instructional instrument in the sense proposed by Andrade [18]. It publicizes both how mathematical modelling is developed, as well as to evaluate mathematical modelling processes throughout the different educational stages, from 3 to 18 years. The theoretical references that guided the construction of the rubric include contributions from Blum and Leiβ [30] about the understanding of modelling processes as a cycle that takes place in different scenarios in which modelling is put into practice. In addition, the guidelines of various international organizations were considered, mainly the Guidelines for Assessment And Instruction In Mathematical Modelling Education (COMAP & SIAM, [32]), as well as the CCSSM (National Governors Association Center for Best Practices and Council of Chief State School Officers [2]) and some important NCTM curricular guidelines [3–5]. In addition, previous literature on the subject was considered, in particular Frejd's [17] meta-analysis of the main proposals for evaluating mathematical modelling.

The external validation of the rubric by eight experts and the internal validation using the CVR′ index (Tristán-López [48]) led to important changes, both in the selection, as well as in the reformulation of some of the items. For example, in the understanding phase, the item "Understands the statement of the problem to be solved" was eliminated as a result of not meeting the required CVR values, as it is considered by experts to be too general an item. In the structuration phase, the item "Proposes solution strategies" was eliminated, as it is not relevant in this phase of the cycle. In the mathematization phase, among others, the item "Uses his or her mathematical knowledge" was modified to "Replaces the real elements with mathematical objects", as in this phase, the focus is on the translation from the real world to the mathematical world. In the mathematical work phase, the item "Obtains an initial mathematical model as a consequence of previous work" was incorporated, as in this phase, the mathematical work results in what is at least an initial model that will be improved throughout the process.

In the interpretation phase, the item "Reflects on the coherence of the mathematical results obtained" was improved to "Checks the coherence of the mathematical solution applied to the initial real context", as it is important that it is explicitly mentioned that the results must be interpreted in the initial real context of the problem. In the validation phase, the item "Check the validity of the results obtained" was modified to "Evaluate whether the obtained model provides a partial or total solution to the initial problem", as we consider it important that the item be more specific and assess the strengths and/or limitations of the model.

Finally, in the presenting phase, some items were incorporated, such as "In the case of using technology in one or more phases of the process, they clearly state when, how and what it was used for", as it is highly probable that students make use of technology in one or more phases of the process, so it is very important that they communicate how they used it. In addition, some items were merged—a decision made fundamentally for two reasons: (1) because some items can be used at more than one educational level and (2) to avoid the instrument becoming cumbersome, extensive and difficult to use.

Additionally, the rubric can be used by teachers as an evaluation tool in terms of the modelling processes that students develop when performing an activity of this type, which is framed according to the concept of formative assessment, as both students and teachers can identify the strengths and weaknesses that may arise in the work of mathematical modelling, thereby improving processes on an ongoing basis. In addition, the rubric allows teachers to generate strategies that strengthen mathematical competence in the application of mathematical knowledge and reasoning; although the rubric does not include a quantitative assessment, it is left to the teacher's discretion to assign an assessment to each of the elements.

On the other hand, this rubric allows for the implementation of mathematical modelling activities in the classroom, which makes it possible to record the students' work in audio, video and written reports and use the rubric for the respective analysis.

This study is subject to two main limitations: (1) Mathematical modelling is a complex process, and the different modelling cycles contribute to its simplification. For this reason, we are aware that the boundaries between each of the phases of the modelling cycle are quite blurred and that on some occasions, it will surely be difficult to determine with complete accuracy in which phase of the cycle the various productions of the students fall. However, the idea is to make this proposal available to teachers and the scientific community so that it can benefit from the contributions of educational practice. (2) Furthermore, the rubric has only been used in the modelling environments presented in Section 4. However, this is one of the aspects that is present on our research agenda, with the aim of discovering more detail regarding the modelling processes, as well as fine-tuning the instrument.

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Appendix A

Table A1. CVR' indices for instrument items.

CVR' and CVI Calculation						
Item	**Essential**	**Useful**	**Not Necessary**	**CVR'**	**Acceptable**	**CVI**
1.1.a.	6	2	0	0.75	X	
1.2.a.	6	1	1	0.75	X	
1.3.a.	6	2	0	0.75	X	
1.1.b.	6	2	0	0.75	X	
1.2.b.	8	0	0	1.00	X	
1.3.b.	6	2	0	0.75	X	
1.1.c.	3	1	4	0.38		
1.2.c.	6	1	1	0.75	X	
1.3.c.	7	1	1	0.88	X	
1.1.d.	3	1	4	0.38		
1.2.d.	6	2	0	0.75	X	
1.3.d.	7	1	1	0.88	X	
2.1.a.	2	1	5	0.25		
2.2.a.	6	2	0	0.75	X	
2.1.b.	1	2	5	0.13		
2.2.b.	4	1	3	0.50		
2.1.c.	4	1	3	0.50		

Table A1. *Cont.*

CVR′ and CVI Calculation						
Item	**Essential**	**Useful**	**Not Necessary**	**CVR′**	**Acceptable**	**CVI**
2.2.c.	1	2	5	0.13		
2.1.d.	6	1	1	0.75	X	
2.2.d.	1	2	5	0.13		
2.3.d.	6	1	1	0.75	X	
3.1.a.	2	2	4	0.25		
3.2.a.	1	2	5	0.13		
3.1.b.	1	3	4	0.13		
3.2.b.	5	1	2	0.63	X	
3.1.c.	6	1	1	0.75	X	
3.2.c.	6	2	0	0.75	X	
3.1.d.	5	0	3	0.63	X	
3.2.d.	6	2	0	0.75	X	
3.3.d.	5	1	2	0.63	X	36.63/47 = 0.78
4.1.a.	1	3	4	0.13		
4.1.b.	6	1	1	0.75	X	
4.1.c.	6	2	0	0.75	X	
4.2.c.	5	2	1	0.63	X	
4.1.d.	6	1	1	0.75	X	
4.2.d.	5	1	2	0.63	X	
5.1.a.	5	3	0	0.63	X	
5.1.b.	6	1	1	0.75	X	
5.1.c.	6	1	1	0.75	X	
5.1.d.	5	1	2	0.63	X	
6.1.a.	7	1	0	0.88	X	
6.2.a.	5	1	2	0.63	X	
6.1.b.	7	1	0	0.88	X	
6.2.b.	8	0	0	1.00	X	
6.1.c.	7	1	0	0.88	X	
6.2.c.	8	0	0	1.00	X	
6.3.c.	7	1	0	0.88	X	
6.1.d.	7	1	0	0.88	X	
6.2.d.	8	0	0	1.00	X	
6.3.d.	6	2	0	0.75	X	
7.1.a.	7	1	0	0.88	X	
7.2.a.	7	1	0	0.88	X	
7.1.b.	7	1	0	0.88	X	
7.2.b.	6	1	1	0.75	X	
7.1.c.	7	1	0	0.88	X	
7.2.c.	6	1	1	0.75	X	
7.1.d.	6	2	0	0.75	X	
7.2.d.	6	1	1	0.75	X	
7.3.d.	6	1	1	0.75	X	
Sum of acceptable items				**36.63**	**47**	

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