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Ecuadorian children's repeating patterning abilities and its association with early mathematical abilities

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Abstract
Young children's early repeating patterning abilities are important foundations for their later mathematical development. Prior studies on young children's repeating patterning abilities have been conducted exclusively in developed countries differing in economic, societal, and educational characteristics from developing countries. In this study, we analyzed the performance and the errors of 4- to 5-year-olds on three repeating patterning tasks, and we investigated the association between children's repeating patterning abilities and their early mathematical abilities in a developing country, Ecuador. A total of 60 preschoolers and 56 kindergartners completed a repeating patterning test and an early mathematics test at the end of the school year. Results first indicated rather low repeating patterning performances of both preschoolers and kindergartens, with large inter-individual differences in these performances. Kindergartners had significantly better patterning abilities than preschoolers. Second, error analyses indicated that children made different types of errors when solving the patterning tasks. The majority of errors were pattern-related errors, with preschoolers making more errors that were not pattern related than kindergarteners. Third, children's patterning abilities were positively associated with early mathematical abilities in both grades. These results complement current theoretical models of young children's mathematical development and inform educational policy and practices in this domain for developing countries.

Keywords Repeating patterns · Early mathematical abilities · Preschool · Kindergarten · Developing country

Introduction

From a very young age, children are exposed to repeating patterns in a variety of daily life situations (e.g., night follows day; alternating colored stripes on clothes). Early experiences

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with repeating patterns are considered central for mathematics learning (Sarama and Clements 2009). Prior research has reported large inter-individual differences in children's repeating patterning performance already before the start of formal schooling (Wijns et al. 2019b; Lüken 2012; Rittle-Johnson et al. 2013, 2017). These individual differences in repeating patterning are associated with individual differences in children's concurrent and later mathematical development (Wijns et al. 2019b; Lüken et al. 2014; Nguyen et al. 2016; Papic et al. 2011; Rittle-Johnson et al. 2017, 2019; Zippert et al. 2019).

Previous studies on repeating patterning abilities and their association with children's concurrent and later mathematical development have been conducted on children in developed countries in Europe (Wijns et al. 2019a; Lüken 2012), USA (Fyfe et al. 2017; Zippert et al. 2019), and Australia (Papic et al. 2011). The generality of current insights into children's patterning abilities and their association with mathematical abilities for also developing countries differing in societal and educational characteristics from developed countries remains an open question. In this study, we aimed at addressing this limitation by examining 4- to 5-year-olds' repeating patterning abilities, with special attention for the types of errors they make when solving repeating patterning tasks, and the association between children's repeating patterning abilities and early mathematical abilities in a developing country, namely, Ecuador (United Nations 2016).

Repeating patterns

Repeating patterns have been defined as linear patterns that have a unit that repeats (Rittle-Johnson et al. 2013). This unit of repeat contains a group of individual items (Papic and Mulligan 2007) and can vary in the number and complexity of items depending on different attributes such as color, shape, size, dimension, and direction (Papic 2007). For example, the following pattern can be coded as an AB pattern with three repeats: O Δ O Δ O Δ, where O Δ is the repeating unit.

Repeating patterning abilities have been investigated using a variety of tasks (Wijns et al. 2019a; Collins and Laski 2015; Papic et al. 2011; Rittle-Johnson et al. 2013, 2015). These tasks include duplicating a pattern (i.e., making an exact replica of a model pattern), extending a pattern (i.e., continuing a pattern by at least one unit of repeat), generalizing (or abstracting) a pattern (i.e., copying a model pattern using a different set of materials), or identifying the unit of repeat in a pattern (i.e., drawing a pattern from memory using the same number of units as the model pattern). The first two types of tasks, namely, duplicating and extending, are assumed to not (necessarily) require insight into the unit of repeat and can also be solved via superficial strategies, such as a one-to-one appearance matching strategy (Collins and Laski 2015). The last two types of tasks, namely, generalizing and identifying the unit, are assumed to require a deeper understanding of the unit of repeat and the use of relational strategies (Rittle-Johnson et al. 2015; Sarama and Clements 2009).

Development of repeating patterning abilities

Previous studies have documented that young children have a range of repeating patterning abilities that develop systematically from preschool to kindergarten (Lüken 2018; Rittle-Johnson et al. 2013, 2015; Sarama and Clements 2009). In their learning trajectories approach, Sarama and Clements (2009) provide a description on how young children progress in their

abilities to solve repeating patterning tasks. In their model, 4-year-olds are able to interpolate, duplicate, and extend AB repeating patterns. At the age of five, children can extend more complex repeating patterns like ABB. At the age of six, they are able to identify the smallest unit of a pattern. Sarama and Clements' (2009) model is confirmed in the empirical studies of Lüken (2018) and Rittle-Johnson et al. (2013, 2015). Lüken (2018) reported that children's repeating pattern abilities considerably developed between the age of three to five. The author observed that at the age of three, no child was able to copy, interpolate, or extend AB repeating patterns. At the age of four, most children accurately copied, interpolated, and extended AB repeating patterns. They were also more or less able to copy, interpolate, and extend ABC and AAB repeating patterns. Most of them could generalize AB repeating patterns but failed to generalize ABC and AAB repeating patterns. When children were aged 5 years, they made little progression (compared with the previous year) in copying, interpolating, and extending AB, ABC, and AAB repeating patterns, but their performance increased for the AAB generalizing patterns. Rittle-Johnson et al. (2013, 2015) found that 4-year-olds' repeating patterning abilities improved during the preschool year. At the beginning of the school year, these children were able to not only duplicate and extend repeating patterns; about one third of these children was also able to generalize repeating patterns and a few of them could even identify the unit of repeat. By the end of the school year the authors observed a substantial improvement in children's performance on duplicating, extending, and generalizing repeating patterns but not in pattern unit recognition. These studies overall suggest a progression in difficulty levels over the different types of pattern tasks: Duplicate tasks are the easiest to solve, followed by extend tasks, generalize tasks, and, finally, unit identification tasks (Rittle-Johnson et al. 2013, 2015; Sarama and Clements 2009).

More recently, researchers started to analyze the type of errors children make when solving repeating patterning tasks (e.g., Collins and Laski 2015; Rittle-Johnson et al. 2013, 2015). Rittle-Johnson et al. (2013) classified the errors of 4-year-olds on repeating patterning tasks into six categories. The first three categories included pattern-related errors, whereas the last three categories involved non-pattern-related errors: (1) partially correct, defined as producing at least one complete unit of the model pattern; (2) wrong pattern AB, defined as making an AB pattern albeit a wrong one; (3) wrong pattern other, defined as creating a wrong three- or four-element pattern; (4) sort, defined as sorting the given objects by color or shape; (5) random order, defined as making linear sequences of blocks in random order; and (6) off task, defined as using blocks but in a way that is not related to the pattern task. The authors observed that the most common error, made by more than half of the children, was to create a partially correct pattern. About half of the children made sorting, random ordering, and wrong pattern AB errors. In a follow-up study with the same group of children, Rittle-Johnson et al. (2015) compared the errors made by 4-year-olds in two measurement moments (i.e., fall and spring) of the preschool year. The authors reported that the total number of non-pattern-related errors decreased at the second measurement moment (i.e., spring) but the number of pattern-related errors did not differ between the two measurement points, which indicates that children's errors evolve from more non-pattern-related errors to more sophisticated, pattern related errors, across the preschool year. Likewise, Collins and Laski (2015) examined the error types that 3- to 5-year-olds made when solving a modified version of the repeating patterning tasks developed by Rittle-Johnson et al. (2013). The authors

observed that these young children most frequently made pattern-related errors than non-pattern-related errors.

Association between repeating patterning abilities and early mathematical abilities

Different studies highlight the important role of repeating patterning abilities in the development of early mathematical abilities (Lüken 2012; Lüken et al. 2014; Nguyen et al. 2016; Papic et al. 2011; Rittle-Johnson et al. 2017, 2019; Wijns et al. 2019c; Zippert et al. 2019). The studies documenting this association have provided both correlational and causal evidence. Concerning the correlational evidence, cross-sectional studies have reported concurrent associations between 4- and 5-year-olds repeating patterning abilities and mathematical abilities (Wijns et al. 2019a; Rittle-Johnson et al. 2019; Zippert et al. 2019). Moreover, longitudinal studies provide evidence for the predictive association between children's repeating patterning abilities and their later mathematical abilities. A study conducted by Lüken (2012) showed that 6-year-olds' performance on a patterning assessment offered at the start of their formal schooling predicted their performance on a mathematics test at the end of grade 2. Additional longitudinal studies reported that repeating patterning abilities at the start of preschool were unique predictors of early mathematical abilities 7 months later (Rittle-Johnson et al. 2019), 1 year later (Wijns et al. 2019c), at the end of grade 1 (Lüken et al. 2014), and even up to the end of fifth grade (Rittle-Johnson et al. 2017).

Regarding the causal evidence, in a quasi-experimental study, Papic et al. (2011) found that children who followed a patterning intervention in preschool developed more advanced mathematical abilities, including counting, symbolic mapping, and calculation knowledge, at the end of kindergarten than their peers who followed the typical preschool instruction. Overall, these findings indicate that early repeating patterning abilities play a role in the development of children's mathematical abilities.

The present study

As indicated above, cumulative evidence indicates that young children's repeating patterning abilities are associated with their early mathematical development (Lüken et al. 2014; Papic et al. 2011; Rittle-Johnson et al. 2019; Wijns et al. 2019c). However, prior studies on young children's repeating patterning abilities have been conducted only in developed countries, limiting the generalizability of prior results to children from developing countries, as suggested by Warren and Miller (2016). These authors analyzed the repeating patterning abilities of Indigenous Australian 4-year-olds at the start of kindergarten and concluded that these children's patterning competencies did not align with the learning trajectories approach proposed by Sarama and Clements (2009; see above). For instance, for many of their participants, it was less difficult to duplicate than to interpolate a pattern, which contrasts the learning trajectory proposed by Sarama and Clements (2009).

This conclusion aligns with those from other cross-cultural comparison studies in different academic domains that acknowledge the limited generalizability of the results from one cultural context to a different one (e.g., Fuson and Li 2009; Li et al. 2009; Silver 2009). Accordingly, the fact that prior studies on children's repeating patterning development have been conducted only in developing countries, limits current theoretical models of young children's early patterning development, and constrains the educational implications of previous findings on early mathematics education for developing countries.

In addition to this absence of empirical studies on children's repeating patterning abilities in developing countries, previous research primarily focused on children's accuracy when solving repeating patterning tasks and paid hardly any attention to the types of errors they make. This limits our understanding of children's early patterning competencies, as the types of errors provide important additional information on their insights into the domain. We aimed at addressing these two weaknesses by analyzing young children's repeating patterning abilities and errors, and the association between children's repeating patterning abilities and their early mathematical abilities in Ecuador, a developing country (United Nations 2016). As such, we aimed to evaluate the generalizability of current findings on children's repeating patterning development outside the commonly studied countries and to refine current theoretical models on young children's repeating patterning abilities and errors, as well as to provide building blocks for optimizing current mathematics education in developing countries, especially Ecuador.

We formulated two sets of research questions:

1a. How do Ecuadorian preschoolers and kindergartners perform on repeating patterning tasks and what are the types of errors they make when solving these tasks?
1b. Do we observe differences in repeating patterning abilities and the types of errors made when solving repeating patterning tasks, between Ecuadorian preschoolers and kindergartners?
2a. Is there an association between Ecuadorian preschoolers' and kindergartners' repeating patterning abilities and their early mathematical abilities?
2b. Do we observe differences in the association between mathematical abilities and repeating pattering abilities between Ecuadorian preschoolers and kindergartners?

Finally, past research has provided evidence that children's personal and contextual variables such as age, gender, and socio-economic status (SES) are associated to the development of repeating patterning abilities (e.g., Lüken and Sauzet 2020; Rittle-Johnson et al. 2013; Starkey et al. 2004). Lüken and Sauzet (2020) found that age and gender played a significant role in the performance of 3- to 5-year olds in patterning tasks, with older children exhibiting better performances (see also Rittle-Johnson et al. 2013) and girls scoring higher on these tasks. Two other studies, conducted in the USA, with preschool children coming from low, middle, and middle-high SES backgrounds, yielded lower repeating patterning abilities among children from low SES families compared with those from middle- to middle-high SES families (Rittle-Johnson et al. 2013; Starkey et al. 2004). Therefore, we decided to control for these three variables, i.e., age, gender, and SES, in our quantitative analyses.

Socio-economic and educational context in Ecuador

As a developing country, Ecuador considerably differs from developed countries in terms of its general economic, societal, and educational characteristics. With respect to the socio-economic characteristics, about 35% of the Ecuadorian inhabitants live in poverty (INEC 2019), around 39,278 Ecuadorian children were out of school in 2016 (UNESCO 2019), and 10.62% of the population is illiterate (Ministerio de Educación 2017). This overall lower socio-economic status has serous constraints for the organization of education as well, as the financial resources available for education are limited as well (cf. Lampert 2003). A recent study on the quality of Ecuadorian kindergarten education (Bojorque et al. 2018) revealed that the quality of early

mathematics instruction in Ecuador is characterized by a direct teaching approach, with children solving paper-and-pencil worksheets and hardly any opportunities for the children to initiate activities themselves or reflect on their own strategies. Compared with USA classrooms, the quality of instruction in Ecuadorian classrooms was relatively low (Bojorque et al. 2018).

Turning to the formal organization of Ecuadorian education, compulsory education starts at the age of 5–6 years. The educational system comprises three levels: beginning level (for children up to 5 years), basic education (for children aged 5 to 14 years), and high school (for students aged 15 to 17 years). Beginning level consists of two sub-levels and is not compulsory: Beginning level 1 for children up to 3 years and Beginning level 2 or preschool for children aged 3 to 5 years. Kindergarten education corresponds to the first year of basic education (5- to 6-year-olds). Both preschools and kindergartens are regulated by the Ministry of Education, which issues a national mandatory curriculum. In the domain of mathematics, specifically with respect to patterns, the national preschool curriculum includes one skill, namely, extend and duplicate simple repeating patterns with concrete objects and graphic representations (Ministerio de Educación 2014). Likewise, the kindergarten curriculum includes one skill related to patterns, namely, describe and reproduce repeating patterns with surrounding objects by color, shape, size, length, or with silhouettes of geometric figures, sounds, and movements (Ministerio de Educación 2016).

Method

Participants

Participants were 116 Ecuadorian 4- to 5-year-olds. The sample consisted of 60 preschoolers (mean age 5 years 1 month (SD = 3.2 months)) and 56 kindergartners (mean age 5 years 11 months (SD = 3.4 months)) coming from six different schools of the three main school types of Ecuador (two public urban, two public rural, and two private schools). We randomly selected 20 preschoolers and 20 kindergartners per school type (10 children per school). Parents' informed consents were gathered from all participating children. Four kindergartners were absent during the data collection, reducing the sample of 120 children to 116 children.

Table 1 presents the number of children, their age, and SES per grade. Children's SES was constructed on the basis of the educational level of the mother (e.g., Aunio and Niemivirta 2010). The level of maternal education was organized into three categories, representing the highest educational level of the mother: (1) basic education, (2) high school, and (3) higher education. As displayed in Table 1, about one third of the participants' mothers attended only

Table 1 Number of children, age, and SES per grade

				Mean age (SD)	Number of children per SES category*		
Grade	Boys	Girls	Total		1	2	3
Preschool	32	28	60	5 years 1 m (3.2 m)	21	16	23
Kindergarten	29	27	56	5 years 11 m (3.4 m)	26	11	19

*1 = highest educational level of the mother is basic education, 2 = highest educational level of the mother is high school, 3 = highest educational level of the mother is higher education

basic education, and more than half of them did not obtain a higher education degree, which is in line with the general low SES level of the population in Ecuador.

Materials

Repeating patterning abilities

To assess children's repeating patterning abilities, we used the repeating patterning test developed by Wijns et al. (2019a). This instrument follows a well-balanced design consisting of three tasks with repeating patterns, i.e., extend, generalize, and identify the unit of repeat. In the *extend* task, children are asked to point out (from a group of four figures) the figure that best completes the series of a given pattern. In the *generalize* task, children are asked to copy a model pattern, using paper figures that differ in color and shape from the model pattern. Children are given a strip of paper and a sufficient number of paper figures to create at least three units of the model pattern. In the *identify the unit of repeat* task, children have to look at the pattern for a period of 6 to 12 s (1 s for each element), try to remember it, and reconstruct it. Again, children are given a strip of paper and a sufficient number of paper figures to create at least three units of the requested pattern. Each of these three tasks comprise six repeating pattern items (i.e., AB, AAB, ABC, AABB, AABC, and ABCD), leading to a total of eighteen items. The tasks are scored dichotomously: a score of 1 assigned for a successful completion of the task and a score of 0 for an unsuccessful completion (maximum score = 18; see Wijns et al. 2019a, for further information). To establish inter-rater reliability, a second rater coded 10% of the scoring forms. Agreement was high ($K = .92$, $p < .001$).

The internal consistency of the overall repeating pattern measure in our sample was acceptable for both preschoolers ($\alpha = .69$) and kindergartners ($\alpha = .71$). However, as shown in Table 2, it was found that only one task (i.e., extend repeating patterns) in kindergarten was of sufficient internal consistency. The values for the other tasks in both preschool and kindergarten ranged from poor to reasonable reliability levels. We will therefore not report any analysis at the individual task level.

We coded children's errors using three task-specific error analysis schemes, based on an adapted version of the scheme developed by Rittle-Johnson et al. (2013). For the extend task, the child is required to select one answer from four options: correct, error based on color, error based on shape, and other error. Accordingly, we distinguished among three categories of errors for this task: (1) errors based on color, wherein the child selects the correct shape, but the wrong color; (2) errors based on shape, wherein the child selects the correct color, but the wrong shape; and (3) other errors, wherein the child selects an option with a wrong element that is part of the pattern; shape and color of this element are incorrect. The first two error types indicate some pattern awareness, demonstrated by choosing an element that completes a pattern focusing on either shape or color (although the element is not part of the model pattern). The last error type is assumed to also involve some pattern awareness, this is,

Table 2 Cronbach's alpha for the three repeating pattern tasks in preschool and kindergarten

Cronbach's alpha	Preschool	Kindergarten
Extend	.39	.74
Generalize	.54	.55
Identify the unit of repeat	.64	.51

understanding that the element that completes the pattern needs to be part of the model pattern. For the generalize task, we distinguished among four error categories: (1) partially correct, (2) wrong pattern, (3) sort, and (4) other. The first two error types, assumed to reflect pattern-related errors, reveal a partial pattern understanding, demonstrated by producing at least one complete unit of the model pattern (partially correct), and creating a pattern different from the model pattern (wrong pattern). The last two error types, assumed to be non-pattern-related errors are less sophisticated errors that do not involve creating patterns, demonstrated by sorting the figures (sort), or placing the figures in a random arrangement (other). Finally, the categories related to the identify task involve the same four error types as in the generalize task, and add an additional pattern-related error type, namely, order error, defined as producing a correct pattern, but with elements within the unit in a wrong order (see Tables 4, 5, and 6 for a description and illustration of each error type). To assess reliability, two raters coded children' responses on 10% of the scoring forms. Cohen's kappa ($K = .93$, $p < .000$) indicated high agreement between the two raters.

Mathematical abilities

Children's early mathematical abilities were assessed via the Test of Early Number and Arithmetic (TENA; Bojorque et al. 2015). The TENA is a test based on the Ecuadorian standards for kindergarten number and arithmetic. It is composed of 54 items divided over nine subscales, namely, (1) Quantifiers, (2) One-to-one correspondence, (3) Order relations more than/less than, (4) Counting, (5) Quantity identification and association with numerals, (6) Ordering, (7) Reading and writing numerals, (8) Addition, and (9) Subtraction. Each subscale consists of six items ordered according to a progressive level of complexity. The test has two parts: an individual part with 29 items that mainly require an oral response (12 tasks require the use of small blocks for children to manipulate or for the examiner to present the task), and a collective part with 25 items that require the use of a paper and pencil. The items are scored dichotomously, with a score of 1 indicating a correct answer and a score of 0 an incorrect one (maximum score = 54). A study focusing on the psychometric qualities of the TENA (Bojorque et al. 2015) demonstrated high overall reliability (Cronbach's $\alpha = .91$).

Procedure

The instruments were administered by trained research assistants at the end of the school year. All participating children received the tests in two sessions, in a separate and quiet room in their school. In the first session, children were individually administered the repeating patterning test. During the second session, children first received the individual part of the early mathematics test and immediately afterwards the collective part.

Results

Ecuadorian preschoolers' and kindergartners' repeating patterning performance

Descriptive statistics indicated rather low repeating patterning performances of both preschoolers and kindergartens (RQ1a). Preschoolers were able to accurately solve about one third of the items ($M = 6.52$, $SD = 3.14$), whereas kindergarteners accurately solved about half

of the items ($M = 9.95$, $SD = 3.36$). The data showed large inter-individual differences in children's performances, with some children able to correctly solve only two (in preschool) or three (in kindergarten) items and others solving up to 16 of the 18 items correctly.

To analyze whether there are differences in repeating patterning abilities between preschoolers and kindergartners (RQ1b), we conducted a regression analysis with children's score on the repeating patterning abilities test as dependent variable and grade (preschool/kindergarten) as independent variable. The control variables age, gender, and SES were also entered in the model as independent variables. The results, displayed in Table 3, indicated that after controlling for the three control variables, there was a significant effect of grade on children's repeating pattering performance, with kindergartners scoring significantly higher than preschoolers.

Error analysis

In addition to children's performance on the repeating patterning test, we analyzed their errors when solving the three repeating patterning tasks (second part of RQ1a) and compared the errors made by preschoolers and kindergartners (second part of RQ1b). We calculated the percentage of preschoolers' and kindergartners' errors per type and per task. In this analysis, we included children's errors on all six items per task. Children who did not make any error on a task were excluded from the analysis. Applying this principle, the number of children that were excluded from the error analysis of the extend, generalize, and identify task was, respectively, eleven (one preschooler and ten kindergartners) five (two preschoolers and three kindergartener), and two (one preschooler and one kindergartner).

As displayed in Table 4, overall, the highest percentage of errors made by children belonged to the "other" category. A smaller percentage of errors fell under the category "only color," followed by "only shape" errors. For preschoolers the errors were more or less equally spread over the three categories, with slightly more "other" errors, whereas for kindergartners more than half of the errors were "other" errors. Pearson's chi-squared analyses indicated that the percentage of "other" errors made by kindergarteners was significantly higher than those made by preschoolers, revealing that kindergarteners demonstrate higher understanding that the chosen element needs to be part of the model pattern.

Regarding the generalize task, as showed in Table 5, the "wrong pattern" errors were the most common, followed by "other" errors and "partially correct" errors, and finally, "sort" errors. Chi-squared analyses indicated no significant differences between the errors made by preschoolers and kindergartners in any category. Interestingly, more than two thirds of the

Table 3 Regression analysis for the influence of children's grade on their repeating patterning abilities

Predictors	*B*	*SE B*	B	*p*
Intercept	7.63	0.80		.000
Age[a]	0.12	0.09	0.11	.164
Gender	− 0.03	0.57	− 0.01	.958
SES	1.25	0.32	0.30	.000
Grade (preschool/kindergarten)	− 3.67	0.57	− 0.50	.000

$R^2 = .30$

[a] Given that the independent variable age is highly related to grade (preschoolers are, on average, 12 months younger than kindergartners), scores for this variable were centered around the mean age of children's grade level

Table 4 Number and percentage of different error types on the extend task by grade

Examples for ABC pattern 

Error	*Description*	*Example*	*Preschool*	*Kindergarten*	*Total*
Only shape	Complete the pattern focusing on shape only		68 (28%)	21 (14%)**	89
Only color	Complete the pattern focusing on color only		77 (31%)	37 (24%)	114
Other	Other element of the pattern (not color only, not shape only)		99 (41%)	96 (62%)**	195
Total			244 (100%)	154 (100%)	398

Difference between preschoolers and kindergartners based Pearson's chi-squared test: ** $p < .01$

errors were pattern-related errors (i.e., partially correct and wrong pattern) both in preschool (70%) and in kindergarten (74%).

With respect to the identify the unit task, as presented in Table 6, in preschool, the "other" errors were the most common errors. The "wrong pattern," "partial correct," and "order" errors were almost equally distributed, whereas the "sort" errors were the least common. In kindergarten, except for the "sort" errors—actually, none of the children made a "sort" error—the errors made by children were somewhat equally spread over the other four categories. Chi-

Table 5 Number and percentage of different error types on the generalize task by grade

Examples for ABC pattern 

Error	*Description*	*Example*	*Preschool*	*Kindergarten*	*Total*
Partially correct	At least one full unit of the model pattern		39 (19%)	29 (20%)	59
Wrong pattern	Two correct units of other pattern		90 (44%)	71 (48%)	161
Sort	Sorts elements on the basis of color and/or shape		7 (3%)	2 (1%)	9
Other	Other errors, not possible to classify as one of the other types of errors (e.g. random order)		70 (34%)	45 (31%)	124
Total			206 (100%)	147 (100%)	353

Pearson's chi-squared analyses indicated no differences between preschoolers and kindergarteners in any of the four categories

squared analyses indicated that kindergarteners made significantly more "order" errors than preschoolers, whereas preschoolers made significantly more "other" errors than kindergartners. Finally, 57% of the errors in preschool and 72% of the errors in kindergarten were pattern-related errors, indicating that (a) the majority of errors made by Ecuadorian children were pattern-related errors and that (b) kindergarteners made more pattern-related errors than preschoolers.

Association between repeating patterning abilities and early mathematical abilities

To evaluate whether there is an association between Ecuadorian preschoolers' and kindergartners' repeating patterning abilities and their early mathematical abilities (RQ2a) and to explore whether the association between children's repeating patterning performances and their early mathematical scores differed in view of grade (preschool vs. kindergarten; RQ2b), we first computed correlations between children's repeating patterning test scores and their scores on the TENA, as well as correlations between these two variables and the control variables age, gender, and SES. As shown in Table 7, children's performance on the repeating patterning test significantly correlated with TENA scores both in preschool and in kindergarten. Additionally, preschoolers' and kindergartners' repeating patterning test scores were significantly correlated with children's SES background, but not with their age and gender. Finally, children's TENA scores significantly correlated with their SES background only in preschool and with children's age only in kindergarten.

Table 6 Number and percentage of different error types on the identify the unit of repeat task by grade

Examples for ABC pattern 

Error	*Description*	*Example*	*Preschool*	*Kindergarten*	*Total*
Partially correct	At least one full unit of the model pattern		48 (20%)	32 (21%)	80
Wrong pattern	Two correct units of other pattern		50 (21%)	31 (20%)	81
Order	Correct pattern, but elements within the unit in wrong order		39 (16%)	48 (31%)**	87
Sort	Sorts elements on the basis of color and/or shape		2 (1%)	0 (0%)	2
Other	Other errors, not possible to classify as one of the other types of errors (e.g. random order)		104 (43%)	43 (28%)**	147
Total			243 (100%)	154 (100%)	397

Difference between preschoolers and kindergartners based Pearson's chi-squared test: $^{**}p < .01$

Next, we conducted a regression analysis with TENA as the dependent variable, children's grade, repeating patterning abilities, and the interaction variable grade * repeating patterning abilities as independent variables. The control variables age, gender, and SES were also included in the model as independent variables. The results, displayed in Table 8, indicated that children's grade was significantly associated to their early mathematical abilities. Based on the descriptive data reported above, this finding reveals that kindergartners had better mathematical abilities than preschoolers. There was also a significant positive effect of children's repeating pattering abilities on their early mathematical abilities. We did not observe a significant interaction between grade and repeating patterning abilities, indicating that the association between patterning abilities and early mathematical abilities was not different in preschool compared with kindergarten.

Discussion

Recent investigations point to the pivotal role of repeating patterning abilities in children's early as well as later mathematical development (e.g., Lüken et al. 2014; Nguyen et al. 2016; Papic et al. 2011; Rittle-Johnson et al. 2019; Wijns et al. 2019c; Zippert et al. 2019). Unfortunately, current insights into children's repeating patterning abilities and errors and their association with mathematical abilities in developing countries are limited. The present study addressed this gap by investigating—for the first time in a developing country—4- to 5-year-olds' repeating patterning abilities and errors when solving three repeating patterning tasks, and by examining these children's repeating patterning abilities in association with their early mathematical abilities. A total of 60 Ecuadorian preschoolers and 56 kindergartners completed a repeating patterning test and an early mathematics test. The results indicated a rather low performance of Ecuadorian children on the repeating patterning test, with kindergartners performing significantly better than preschoolers. In general, children made a variety of error types when solving the repeating patterning test; the majority of these errors were pattern-related errors, indicating that although they perform generally low in the test, they seem to have started to develop a basic understanding of repeating patterns. Compared with preschoolers, kindergarteners made less non-pattern-related errors in the extend and identify the unit tasks. Importantly, children's repeating patterning performance was associated with their early mathematical abilities, in both preschool and kindergarten. In this concluding section, we discuss our major findings and reflect on their theoretical and educational implications.

Table 7 Spearman correlations between repeating patterning abilities, TENA, age, gender, and SES in preschool and kindergarten

	Repeating patterns	TENA	Age	Gender	SES
Preschool					
1. Repeating patterns	-	.43**	.09	− .02	.35**
2. TENA		-	.10	.13	.50**
Kindergarten					
1. Repeating patterns	-	.44**	.16	− .02	.32*
2. TENA		-	.31*	.02	.19

*$p < .05$, ** $p < .01$

Table 8 Regression analysis for the influence of children's grade on the association between patterning abilities and early mathematical abilities

Predictors	*B*	*SE B*	B	*p*
Intercept	24.66	3.21		.000
Age [a]	0.30	0.20	0.10	.148
Gender	1.34	1.33	0.07	.316
SES	2.30	0.81	0.21	.005
Grade (preschool/kindergarten)	− 9.39	3.71	− 0.48	.013
Repeating patterns	0.79	0.30	0.30	.009
Grade*repeating patterns	0.12	0.41	0.05	.767

Adjusted R^2 = .47

[a] Given that the independent variable age is highly related to grade (preschoolers are, on average, 12 months younger than kindergartners), scores for this variable were centered around the mean age of children's grade level

Ecuadorian young children's repeating pattering abilities and errors

A first major finding relates to the rather low performance of Ecuadorian preschoolers and kindergartens on the repeating patterning test, with large inter-individual differences in these performances. Compared with a previous study conducted with the same assessment battery and at the same time point in 4-year-olds but coming from a developed country in Europe, i.e., Belgium, Flanders (Wijns et al. 2019a), Ecuadorian preschoolers exhibited relatively low repeating patterning abilities. Whereas Flemish 4-year-olds were able to correctly solve about half of the items, Ecuadorian 4-year-olds accurately responded to (only) about one third of the items. Ecuadorian children's repeating patterning abilities increased during the kindergarten period, with better repeating patterning abilities in kindergartners than in preschoolers. This result is consistent with prior findings indicating that young children's repeating patterning abilities considerably develop over time (Lüken 2018; Papic et al. 2011; Rittle-Johnson et al. 2013; Sarama and Clements 2009).

Furthermore, Ecuadorian 4- to 5-year-olds made a variety of error types when solving the repeating patterning test, which range from less sophisticated, non-pattern-related errors (such as sorting or making linear sequences of elements in random order) to more sophisticated, pattern-related errors (such as creating a partially correct pattern, producing a different pattern, or changing the order of the elements in a pattern). Interestingly, the majority of errors made by these young children were pattern-related errors revealing that despite the poor performances on the repeating patterning test, children had started to develop some initial pattern understanding. Kindergarteners made less non-pattern-related errors in the extend and identify the unit tasks than preschoolers, suggesting that Ecuadorian children's insight into patterns develops between the age of four and five. Overall, our error analyses are consistent with findings from previous research indicating that prior to formal schooling, young children already start to develop an understanding of repeating patterns (Collins and Laski 2015; Lüken 2018; Lüken and Sauzet 2020; Rittle-Johnson et al. 2013, 2015) and that error types made by these young children when solving repeating patterning tasks are mainly pattern-related errors (Collins and Laski 2015; Rittle-Johnson et al. 2013, 2015). Moreover, the differences in the types of the errors made by preschoolers and kindergartners support previous findings indicating that patterning errors become more sophisticated with age (Rittle-Johnson et al. 2015). Taken together, our results indicate that previous findings on the development of repeating patterning abilities and errors, conducted in developed countries, can be generalized

to children in developing countries. However, it is important to note here that the cross-sectional design of our study does not allow to draw definitive conclusions about the developmental trajectories of children in developing versus developed countries. These first findings on young children's development of repeating patterning abilities and errors therefore need to be replicated in future longitudinal studies that follow Ecuadorian children's developmental trajectories from preschool to kindergarten and beyond.

Despite these overall similarities in children's early patterning development, the low performances exhibited by Ecuadorian children on the repeating patterning test are of great concern as previous studies conducted in developed countries pointed to the pivotal role of early repeating patterning abilities in concurrent and later mathematical development (e.g., Lüken 2012; Nguyen et al. 2016; Rittle-Johnson et al. 2019; Wijns et al. 2019a). The difference in repeating patterning performance between Ecuadorian preschoolers and their peers coming from developed countries might be explained in at least three ways. The first explanation relates to the opportunity gaps imposed on lower SES families, including less stimulating home environments, which have been associated with children's mathematical knowledge (Mullis et al. 2012). Given the generally lower SES of the Ecuadorian population (INEC 2019), which is also reflected in the composition of our sample, it might be that Ecuadorian children lack sufficient, and sufficiently high-quality, early home experiences and support that contribute to the development of their repeating patterning abilities. Prior survey and observational studies conducted in developed countries, such as USA, presented evidence that parents provided some support of early mathematical development via numeracy, spatial, and patterning activities (Hart et al. 2016; Lefevre et al. 2009; Rittle-Johnson et al. 2015; Zippert and Rittle-Johnson 2018). Future studies should focus on investigating whether young Ecuadorian children are also receiving some patterning experiences and support at home and, if so, determine whether those home experiences contribute to the development of their mathematical abilities in general and their patterning abilities in particular.

A second explanation for the overall weak repeating patterning performance of Ecuadorian preschoolers compared with their peers from developed countries refers to the rather low quality of early mathematics education in Ecuador. As outlined above, Bojorque et al. 2018 reported that the quality of mathematics instruction in Ecuadorian kindergarten classrooms tended to be rather low, with mainly whole-class and teacher-centered instruction, characterized by primarily completing written worksheets and limited individual interactions or thought-provoking discussions. This low-quality instruction is probably not beneficial for children's learning of mathematics concepts and skills in general and patterning abilities in particular. Thus, it is important that future studies address the quality of early mathematics instruction in developing countries, by evaluating the effectiveness of intervention programs for increasing children's early mathematics development in general (cf. Building Blocks; Bojorque et al. 2018) and patterning abilities in particular (cf. Pattern and Structure Mathematics Awareness Program [PASMAP]; Mulligan et al. 2020).

A third explanation is that we used formal, "school-like" tasks to evaluate Ecuadorian young children's repeating patterning abilities. Consequently, it is possible that we were not able to capture some informal experiences and knowledge about repeating patterns developed by these young children. Therefore, additional research that uses other measures of patterning abilities—such as less formal, play-based measures or measures that tap culturally appropriate activities or artifacts involving patterns (e.g., Warren and DeVries 2009)—is also needed. The use of those measures might be more appropriate to reveal Ecuadorian young children's greater patterning abilities.

Association between repeating patterning abilities and early mathematical abilities

A second major finding of our study is that children's repeating patterning abilities were associated with their early mathematical abilities, both in preschool and in kindergarten. This result is in line with findings from previous studies conducted in developed countries that reported positive associations between 3- and 5-year-olds' repeating patterning abilities and their early mathematical abilities (e.g., Wijns et al. 2019a; Rittle-Johnson et al. 2019). Although the association between repeating patterning abilities and early mathematical abilities reported in this study held after controlling for children's age, gender, and SES, it might be that other relevant variables, such as working memory capacity, are explaining this association. Past research in developed countries has revealed that young children's working memory plays a significant role in their patterning performance but does not influence the association between repeating patterning abilities and early mathematical abilities (Collins and Laski 2015; Fyfe et al. 2017; Rittle-Johnson et al. 2013; Zippert et al. 2019). However, it remains to be evaluated whether these findings hold for developing countries too.

From a theoretical viewpoint, our findings provide the first important evidence for the generalizability of findings from prior studies conducted in developed countries on the association between repeating patterning abilities and mathematical abilities, and thus, contribute to current models of young children early mathematical development by pointing to the universal nature of this association. Despite the differences in young children' economic, societal, and educational contexts in developed and developing countries (i.e., Ecuador), in both groups of children repeating patterning abilities are associated with their early mathematical abilities. As such, our findings suggest that the same mechanisms underlie the developmental association between repeating patterning abilities and early mathematical abilities, across different societal and educational contexts. In order to deepen our theoretical understanding of the mechanisms underlying the development of pattering abilities in relation with early mathematical abilities it is important that future research focuses on developing theoretical models that can help to explain such an association. The recent article of Zippert et al. (2019) offers important building blocks to develop such models. These authors claim that one potential mechanism underlying this association is the consideration and application of rules and regularities in sequences of objects (e.g., identifying that there is a unit of repeat in a pattern, and that this unit of repeat determines the structure for the entire pattern) and in numbers and number sequences (e.g., understanding the rule that the next number in the counting sequence means adding one more). It is a challenge for future studies to evaluate this hypothetical explanation, using a wide range of reliable and valid numerical tasks that focus on the consideration and application of specific rules and regularities. As such, future studies will enable a more comprehensive theory of early mathematics development (cf. Rittle-Johnson et al. 2019). Furthermore, to analyze the association between repeating pattering abilities and early mathematical abilities, the design and implementation of intervention studies that might inform us about potential causal associations are needed, for instance, intervention studies focusing on identifying the underlying structures involved in young children's pattering and mathematical development combined with micro-genetic follow-up of these young children's conceptual understanding of patterns and of mathematics. Such intervention studies can be broadened by adding cross-cultural comparisons that allow to test the universality of those underlying mechanisms.

From an educational viewpoint, the results of the present study provide, for the first time, evidence on young children's repeating pattering abilities and its association to early

mathematics achievement in developing countries. Thus, our findings provide foundational information for policy discussions related to early mathematics education in these countries. Our results also point to the need of implementing educational interventions in Ecuadorian preschools and kindergartens aiming at stimulating children's patterning abilities in view of their further mathematical development. As the present study did not longitudinally follow preschoolers' and kindergartners' further patterning and mathematical development up to the elementary school years and beyond, they can only be hypothetically related to the difficulties that South-American elementary and secondary school students demonstrate in international comparison studies in the domain of mathematics (Mullis et al. 2012; OECD 2014; UNESCO 2015). Fine-grained longitudinal follow-up studies are required to deepen our understanding of the foundations of these difficulties at older ages, following children's early patterning and mathematical development from the start of preschool over their kindergarten year and the first years of formal mathematics education up to their secondary school years. Such long-term descriptive studies of Ecuadorian (and more generally, South-American) children's early mathematics competencies development and later mathematical achievement will provide the necessary insights to design focused intervention studies, addressing the major difficulties in these children's early patterning and later mathematical development.

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References

Aunio, P., & Niemivirta, M. (2010). Predicting children's mathematical performance in grade one by early numeracy. *Learning and Individual Differences, 20*(5), 427–435. https://doi.org/10.1016/j.lindif.2010.06.003.

Bojorque, G., Torbeyns, J., Moscoso, J., Van Nijlen, D., & Verschaffel, L. (2015). Early number and arithmetic performance of Ecuadorian 4-5-year-olds. *Educational Studies, 41*(5), 565–586. https://doi.org/10.1080/03055698.2015.1090302.

Bojorque, G., Torbeyns, J., Van Nijlen, D., & Verschaffel, L. (2018). Ecuadorian kindergartners' numerical development: contribution of SES, quality of early mathematics education, and school type. *Educação e Pesquisa, 44*, 1–21.

Collins, M. A., & Laski, E. V. (2015). Preschoolers' strategies for solving visual pattern tasks. *Early Childhood Research Quarterly, 32*, 204–214. https://doi.org/10.1016/j.ecresq.2015.04.004.

Fuson, K. C., & Li, Y. (2009). Cross-cultural issues in linguistic, visual-quantitative, and written-numeric supports for mathematical thinking. *ZDM Mathematics Education, 41*, 793–808. https://doi.org/10.1007/s11858-009-0183-7.

Fyfe, E. R., Evans, J. L., Matz, L. E., Hunt, K. M., & Alibali, M. W. (2017). Relations between patterning skill and differing aspects of early mathematics knowledge. *Cognitive Development, 44*, 1–11. https://doi.org/10.1016/j.cogdev.2017.07.003.

Hart, S. A., Ganley, C. M., & Purpura, D. J. (2016). Understanding the home math environment and its role in predicting parent report of children ' s math skills. *PLoS One, 11*(12), 1–30. https://doi.org/10.1371/journal.pone.0168227.

INEC (Instituto Nacional de Estadísticas y Censos). (2019). Encuesta nacional de empleo, desempleo y subempleo (ENEMDU). Pobreza y desigualdad. Quito. https://www.ecuadorencifras.gob.ec/pobreza-junio-2019/%0A

Lampert, E. (2003). Educación: visión panorámica mundial y perspectivas para el siglo XXI. *Perfiles Educativos, 25*(101), 7–22.

Lefevre, J. A., Skwarchuk, S. L., Smith-Chant, B. L., Fast, L., Kamawar, D., & Bisanz, J. (2009). Home numeracy experiences and children's math performance in the early school years. *Canadian Journal of Behavioural Science, 41*(2), 55–66. https://doi.org/10.1037/a0014532.

Li, Y., Chen, X., & An, S. (2009). Conceptualizing and organizing content for teaching and learning in selected Chinese, Japanese and US mathematics textbooks: the case of fraction division. *ZDM Mathematics Education, 41*, 809–826. https://doi.org/10.1007/s11858-009-0177-5.

Lüken, M. M. (2012). Young children's structure sense. *Journal fur Mathematik-Didaktik, 33*(2), 263–285. https://doi.org/10.1007/s13138-012-0036-8.

Lüken, M. M. (2018). Repeating pattern competencies in three- to five-year old kindergartners: A closer look at strategies. In C. B. Iliada Elia, J. Mulligan, A. Anderson, & A. Baccaglini-Frank (Eds.), *Contemporary research and perspectives on early childhood mathematics education* (pp. 35–54). Hamburg: Springer.

Lüken, M. M., Peter-Koop, A., & Kollhoff, S. (2014). Influence of early repeating patterning ability on school mathematics learning. In D. Allan, P. Liljedahl, C. Nicol, & S. Oesterle (Eds.), *Proceedings of the 38th conference of the International Group for the Psychology of Mathematics Education and the 36th Conference of the North American Chapter of the Psychology of Mathematics education* (pp. 137–144). Vancouver: PME.

Lüken, M. M., & Sauzet, O. (2020). Patterning strategies in early childhood: a mixed methods study examining 3- to 5- year-old children's patterning competencies. *Mathematical Thinking and Learning.* https://doi.org/10.1080/10986065.2020.1719452.

Ministerio de Educación. (2014). Currículo de educación inicial. Quito.

Ministerio de Educación. (2016). Currículo 2016. Quito.

Ministerio de Educación. (2017). Analfabetismo. *Tasas y absolutos 2006–2016*. Quito.

Mulligan, J., Oslington, G., & English, L. (2020). Supporting early mathematical development through a 'pattern and structure' intervention program. *ZDM Mathematics Education 52*, 663–676. https://doi.org/10.1007/s11858-020-01147-9

Mullis, I., Martin, M., Foy, P., & Arora, A. (2012). *TIMSS 2011 International results in mathematics*. Chestnut Hill, MA: TIMSS & PIRLS International Study Center, Boston College.

Nguyen, T., Watts, T. W., Duncan, G. J., Clements, D. H., Sarama, J. S., Wolfe, C., & Spitler, M. E. (2016). Which preschool mathematics competencies are most predictive of fifth grade achievement? *Early Childhood Research Quarterly, 36*, 550–560. https://doi.org/10.1016/j.ecresq.2016.02.003.

OECD. (2014). *PISA 2012 results in focus: what 15-year-olds know and what they can do with what they know*. Paris: OECD.

Papic, M. (2007). Promoting repeating patterns with young children—more than just alternating colours! *Apm, 12*(3), 8–13.

Papic, M., & Mulligan, J. (2007). Mathematical patterning in early childhood: an intervention study. *Mathematics: Essential Research, Essential Practice, 2*, 591–600 http://oatd.org/oatd/record?record=%22handle:1959.14/215381%22.

Papic, M., Mulligan, J., & Mitchelmore, M. (2011). Assessing the development of preschoolers' mathematical patterning. *Journal for Research in Mathematics Education, 42*(3), 237–268. https://doi.org/10.5951/jresematheduc.42.3.0237.

Rittle-Johnson, B., Fyfe, E. R., McLean, L. E., & McEldoon, K. L. (2013). Emerging understanding of patterning in 4-year-olds. *Journal of Cognition and Development, 14*(3), 376–396. https://doi.org/10.1080/15248372.2012.689897.

Rittle-Johnson, B., Fyfe, E. R., Loehr, A. M., & Miller, M. R. (2015). Beyond numeracy in preschool: Adding patterns to the equation. *Early Childhood Research Quarterly, 31*, 101–112. https://doi.org/10.1016/j.ecresq.2015.01.005.

Rittle-Johnson, B., Fyfe, E. R., Hofer, K. G., & Farran, D. C. (2017). Early math trajectories : Low-income children ' s mathematics knowledge from ages 4 to 11. *Child Development, 88*(5), 1727–1742. https://doi.org/10.1111/cdev.12662.

Rittle-Johnson, B., Zippert, E. L., & Boice, K. L. (2019). The roles of patterning and spatial skills in early mathematics development. *Early Childhood Research Quarterly, 46*, 166–178. https://doi.org/10.1016/j.ecresq.2018.03.006.

Sarama, J., & Clements, D. H. (2009). *Early childhood mathematics education research: Learning trajectories for young children*. New York: Routledge.

Silver, E. A. (2009). Cross-national comparisons of mathematics curriculum materials: what might we learn? *ZDM Mathematics Education, 41*, 827–832. https://doi.org/10.1007/s11858-009-0209-1.

Starkey, P., Klein, A., & Wakeley, A. (2004). Enhancing young children's mathematical knowledge through a pre-kindergarten mathematics intervention. *Early Childhood Research Quarterly, 19*(1), 99–120. https://doi.org/10.1016/j.ecresq.2004.01.002.

UNESCO. (2015). El laboratorio latinoamericano de evaluación de la calidad de la educación (LLECE). *Los aprendizajes de los estudiantes de América Latina y el Caribe*. Santiago - Chile.

UNESCO. (2019). Ecuador. Education and literacy. http://uis.unesco.org/en/country/ec?theme=education-and-literacy. Accessed 9 April 2019.

United Nations. (2016). *World economic situation and prospects 2016*. New York.
Warren, E., & DeVries, E. (2009). Young Australian Indigenous students' engagement with numeracy: actions that assist to bridge the gap. *Australian Journal of Education, 53*(2), 159–175. https://doi.org/10.1177/000494410905300205.
Warren, E., & Miller, J. (2016). Exploring four year old Indigenous students' ability to pattern. *International Research in Early Childhood Education, 1*(2), 42–56.
Wijns, N., Torbeyns, J., Bakker, M., De Smedt, B., & Verschaffel, L. (2019a). Four-year olds' understanding of repeating and growing patterns. *Early Childhood Research Quarterly, 49*, 152–163.
Wijns, N., Torbeyns, J., De Smedt, B., & Verschaffel, L. (2019b). Young children's patterning competencies and mathematical development: A review. In K. Robinson, H. Osana, & D. Kotsopoulos (Eds.), *Mathematical learning and cognition in early childhood*. Cham: Springer.
Wijns, N., Verschaffel, L., De Smedt, B., & Torbeyns, J. (2019c). Which early patterning activities count the most? In NNN (Ed.), *Proceedings of the 43rd Conference of the International Group for the Psychology of Mathematics Education* (Vol. 1, pp. 1–8). Pretoria, South Africa: PME.
Zippert, E. L., & Rittle-Johnson, B. (2018). The home math environment: More than numeracy. *Early Childhood Research Quarterly*, (September). https://doi.org/10.1016/j.ecresq.2018.07.009.
Zippert, E. L., Clayback, K., & Rittle-Johnson, B. (2019). Not just IQ : Patterning predicts preschoolers' math knowledge beyond fluid reasoning not just IQ: patterning predicts preschooler' math knowledge beyond fluid reasoning. *Journal of Cognition and Development, 20*(5), 1–20. https://doi.org/10.1080/15248372.2019.1658587.

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Most relevant publications in the field of Psychology of Education

Bojorque, G., Torbeyns, J., Van Hoof, J., Van Nijlen, D., & Verschaffel, L. (2018). Effectiveness of the building blocks program for enhancing Ecuadorian kindergartners' numerical competencies. *Early Childhood Research Quarterly, 44,* 231–241.
Torbeyns, J., Bojorque, G., Van Hoof, J., Van Nijlen, D., & Verschaffel, L. (2018). Unique contribution of Ecuadorian kindergartners' spontaneous focusing on numerosity to their early numerical abilities. *British Journal of Developmental Psychology, 36,* 299–312.
Bojorque, G., Torbeyns, J., Van Nijlen, D., & Verschaffel, L. (2018). Ecuadorian Kindergartners' numerical development: contribution of SES, quality of early mathematics education and school type. *Educação e Pesquisa*, *44,*1–21.
Bojorque, G., Torbeyns, J., Hannula-Sormunen, M., Van Nijlen, D., & Verschaffel, L. (2017). Development of SFON in Ecuadorian kindergartners. *European Journal of Psychology of Education. 32:3,* 449–462. doi: 10.1007/s10212-016-0306-9
Bojorque, G., Torbeyns, J., Moscoso, J., Van Nijlen, D., & Verschaffel, L. (2015). Early number and arithmetic performance of Ecuadorian 4-5-year-olds. *Educational Studies 41:5,* 565–586. doi: 10.1080/03055698.2015.1090302

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Most relevant publications in the field of Psychology of Education

Gonzales, N., Trelles, C., Mora, J., 2017. Manejo Docente de las Tecnologías de la Información y Comunicación. Innova Research Journal. Cuenca, Ecuador, 61-72.
Gonzales, N., Trelles, C., 2017. La modelación con apoyo de software libre y los cambios en los procesos de aprendizaje en matemáticas en los estudiantes de primero de bachillerato. Revista Educación y Desarrollo Social, 64-73.

Martinez, M., Castillo, P., Trelles, C., Gonzales, N., Calle, E., Ayala, A., ... & Folres, M., 2016. Informe sobre la formación inicial y continua de profesores de matemáticas en el Ecuador. Cuadernos de Investigación y Formación en Educación Matemática, 11-45.

Gonzales, N., 2014, Una forma de aprender expresiones algebraicas manejando conceptos de áreas, Acta Latinoamericana de Matemática Educativa, 27, 1259 - 1265

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Most relevant publications in the field of Psychology of Education

Bakker, M., Torbeyns, J., Wijns, N., Verschaffel, L., & De Smedt, B. (2019). Gender equality in 4- to 5-year-old preschoolers' early numerical competencies. Developmental Science, 22(1), e12718. 10.1111/desc.12718

Verschaffel, L., Rathé, S., Wijns, N., De Smedt, B., & Torbeyns, J. (2018). Young children's early mathematical competencies: the role of mathematical focusing tendencies. Invited plenary lecture. Kristiansand, Norway.

Wijns, N., Torbeyns, J., Bakker, M., De Smedt, B., & Verschaffel, L. (2019). Four-year olds' understanding of repeating and growing patterns and its association with early numerical ability. Early Childhood Research Quarterly, 49, 152–163. 10.1016/j.ecresq.2019.06.004

Wijns, N., Torbeyns, J., De Smedt, B., & Verschaffel, L. (2019). Young children's patterning competencies and mathematical development: a review. In K. Robinson, H. Osana, & D. Kotsopoulos (Eds.), Mathematical learning and cognition in early childhood (pp. 139–161). Cham: Springer International Publishing. 10.1007/978-3-030-12895-1_9

Verschaffel, L., Wijns, N., De Smedt, B., & Torbeyns, J. (2019, August). Four-year olds' spontaneous focusing on patterns. In J. McMullen (Chair), *Beyond SFON: expanding examinations of spontaneous mathematical focusing tendencies*. Symposium conducted at the Earli conference, Aachen, Germany.

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Most relevant publications in the field of Psychology of Education

Verschaffel, L., Luwel, K., Torbeyns, J., & Van Dooren, W. (2009). Conceptualizing, investigating, and enhancing adaptive expertise in elementary mathematics education. *European Journal of Psychology of Education, 24*, 335-359.

Dewolf, T., Van Dooren, W., & Verschaffel, L. (2011). Upper elementary school children's understanding and solution of a quantitative word problem inside and outside the mathematics class. *Learning and Instruction*, *21*, 770-780.

Fernández, C., Llinares, S., Van Dooren, W., De Bock, D., Verschaffel, L. (2012). The development of students' use of additive and proportional methods along primary and secondary school. *European Journal of Psychology of Education*, *27*, 421-438.

Obersteiner, A., Van Dooren, W., Van Hoof, J., & Verschaffel, L. (2013). The natural number bias and magnitude representation in fraction comparison by expert mathematicians. *Learning and Instruction*, *28*, 64-72.

Linsen, S., Verschaffel, L., Reynvoet, B., & De Smedt, B. (2015). The association between numerical magnitude processing and mental versus algorithmic multi-digit subtraction in children. *Learning and Instruction, 35*, 42-50.

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Most relevant publications in the field of Psychology of Education

Torbeyns, J., Verbruggen, S., & Depaepe, F. (in press). Pedagogical content knowledge in preservice preschool teachers and its association with opportunities to learn during teacher training. *ZDM Mathematics Education.*

Torbeyns J., Peters G., De Smedt B., Ghesquière P., & Verschaffel L. (2018). Subtraction by addition strategy use in children of varying mathematical achievement level: a choice/no-choice study. *Journal of Numerical Cognition*, *4*, 215-234.

Torbeyns J., Hickendorff M., & Verschaffel L. (2017). The use of number-based versus digit-based strategies on multi-digit subtraction: 9-12-year-olds' strategy use profiles and task performance. *Learning & Individual Differences*, *58*, 64-74.

Torbeyns J., Peters G., De Smedt B., Ghesquière P., & Verschaffel L. (2016). Children's understanding of the addition/subtraction complement principle. *British Journal of Educational Psychology*, *86*, 382-396.

Torbeyns J., Schneider M., Xin Z., & Siegler R. S. (2015). Bridging the gap: Fraction understanding is central to mathematics achievement in students from three different continents. *Learning and Instruction*, *37*, 5-13.

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