Back-Analysis of Slope Failures by Numerical Techniques

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ABSTRACT

A method of defining strength parameters from slope failures is presented in this study, based on combining finite element and optimization analyses. The strength reduction technique is used to determine the shear strain field. The slip surface is then obtained from shear strain contours by extracting the maximal strains in several vertical lines. This surface is used for the optimization process when comparing to the observed failure surface. The optimization algorithm computes the sensitivity matrix with fix increments to avoid stationary values of the objective function. Two illustrative examples are presented, one considering a simple homogenous slope and the second compose of a two-layered model. The results of the analysis show that the proposed technique is efficient to determining soil parameters from slope failures by retaining the advantages of finite element slope stability analysis.

KEYWORDS: Back-analysis, Soil Parameters, Inverse Modeling, Finite Element Method, Optimization, Nonlinear Least Squares.

INTRODUCTION

When remedial measures are foreseen to be implemented, shear strength properties obtained from back-analysis of slope failures are more reliable than those obtained from laboratory or in-situ tests (Duncan, 1992; Greco, 1992; Leroueil, 2001; Jiang, 2006). The back-calculated values can also be efficiently used to analyze the stability of other slopes with similar characteristics. The actual landslide can be considered as a large-scale field test, where back-analysis is a useful procedure for estimating soil parameters along the slip surface directly. This process helps to overcome some limitations and uncertainties in the use of laboratory and in-situ tests. Moreover, the necessity for determining a representative sample for the testing is avoided. Back-calculated soil properties account for the influences of soil fabric, heterogeneity, fissures, pre-existing shear surfaces, and long-term loading (Tang, 1999).

In conventional back-analysis, using the Mohr-Coulomb failure criterion with parameters c (cohesion) and ϕ (friction angle), the simplest process to back-analyzing soil properties is to use the geometry of the slip surface. This process is based on the fact that for a factor of safety of unity each pair c-ϕ will produce a unique location for the critical slip surface (Wesley and Leelaratnam, 2001; Jiang and Yamagami, 2006). As result, by comparing the actual slip surface with respect to the ones determined from the trial analysis, it is possible to compute the strength parameters at failure. However, this approach cannot provide a unique combination of the parameters c-ϕ since several combinations of strength parameters might give the same slip surface.

Despite, limit equilibrium methods (LEM) lean on assumptions such as: shape and location of the failure surface, constant factor of safety, interslice forces representation, iterative solutions for determining the factor of safety (Ashford and Sitar, 2001; Potts, 2003; Zheng et al., 2005; Duncan and Wright, 2005), until now, back-analysis of slope failures has been performed using these methods. Finite elements methods (FEM) remove these severe assumptions and it is more flexible to account for different slope features and complex geometry. Therefore, a numerical procedure is proposed combining the finite element method and optimization techniques to back-analyzing slope failures.

In order to compare the actual slip surface (slope failure), trials surfaces with different sets of c-ϕ are define by FEM. First, the determination of the critical slip surface (CSS) based on shear contour is discussed. The CSS is traced from the maximal shear strain along vertical lines along the model. This CSS is then compared with the observed slip failure in order to minimize the differences between these two surfaces. Followed, the optimization process is presented, in which the sensitivity matrix is computed with fix increments (step-length) to avoid stationary values. Two illustrative examples are treated highlighting the gains of this approach.

MATERIALS AND METHODS

Determination of the critical slip surface

There is no general criterion to determine the critical slip surface from the collapse finite element analysis. In finite element slope stability analysis, the determination of the failure surface is performed through inspection of the mesh, displacement field, and shear stress field, among the most useful. Comparative analysis between FEM and LEM have shown a good agreement in values for the factor of safety (Rocscience, 2004, Zheng et al., 2005, Cheng, 2006), as well as relatively good agreement between the shear zone in finite elements and the CSS in LEM (Krahn, 2007, Alkasawneh et al., 2008). Although, finite element visual techniques provide a clearly picture of the failure mechanism, for the optimization algorithm the geometrical location of the critical slip surface is necessary.

In this study, the shear strain field obtained from the strength reduction method (SRM) is used to trace the CSS. The slope model is separated in several vertical lines and along each line the maximum strain is located. Additionally, the critical points can be adjusted to a curve by regression analysis. For most of the cases, a third-degree polynomial curve represents closely the critical slip surface of homogenous slopes.

Optimization Algorithm

The basic components of an optimization problem are an *objective function* to be optimized (minimized), a set of *unknowns* (variables) that controls the value of the objective function, and a set of *constraints* which allow the unknowns to take on certain values (constrained optimization). Thereby, the optimization problem consists in finding values of the unknowns that minimize the objective function, while satisfying the constraints (Teughels, 2003). In this context, the objective function is the deviation, the error between experimental (field or laboratory) data and numerical prediction.

The optimization algorithm used in the analysis is a local process (nonlinear least squares, NLLS). This algorithm is implemented in MATLAB toolboxes, from where the forward problem is called. The finite element slope analysis is performed in PLAXIS (finite element code) and the strain field is exported to MATLAB for determining the CSS. Since PLAXIS is a stand-alone application, a MATLAB subroutine is used to mimic the mouse click and move commands needed to input and retrieve data. One of the main advantages of the inverse modeling is that it allows simultaneous calibration of multiple parameters. However, the complex situations encountered in geotechnical applications make the inverse modeling time consuming due to the highly nonlinear system of involved equations. This nonlinearity might due to material and/or geometrical features. Most of the real problems involve nonlinear optimization with complex objective functions for which analytical solutions are not available. Therefore, the numerical algorithm calls the numerical model to compute the gradient of the objective function (sensitivity matrix).

The sensitivity matrix is commonly evaluated using finite differencing to approximate the gradient. In order to avoid the computation of the sensitivity matrix using very small or very large changes in physical parameters, it is proposed that a fix step-length should be used. This change just affects the standard gradient algorithm. This *modified* gradient allows to overcome the problems of very small or very large changes. Moreover, the fix step-length is more engineering soundly. For example, by changing 0.1º in the friction angle there will not be any effect in the system, whereas a 2º change is likely to affect the system.

The objective function for the NLLS is defined as a least square problem where the objective function $f(\mathbf{x})$ is expressed as:

$$f(\mathbf{x}) = \frac{1}{2}\mathbf{r}(\mathbf{x})^T\mathbf{r}(\mathbf{x}) \tag{1}$$

$$\mathbf{r}(\mathbf{x}) = \begin{bmatrix} y_1^* - y_1(\mathbf{x}) \\ y_2^* - y_2(\mathbf{x}) \\ \vdots \\ y_m^* - y_m(\mathbf{x}) \end{bmatrix} \tag{2}$$

where each element of the m-dimensional residual vector $\mathbf{r}(\mathbf{x})$ is a function of the n-dimensional vector ($\mathbf{x}$) with parameters $\mathbf{x}_j$, (j=1,…,n). The residual vector contains the differences between the observed (measured) values ($\mathbf{y}_i^*$) and the computed values derived from numerical analysis ($\mathbf{y}_i(\mathbf{x})$). For example, in a slope failure, the observed values may be the location of the slip surface with the variables being the strength parameters (cohesion and friction angle).

Several methods have been proposed to solve the nonlinear optimization problem. In the Newton-based method, the objective function to be minimized is approximated in the vicinity of the current point $\mathbf{x}_k$ by a model function $m_k(\mathbf{p})$ which is usually a quadratic function:

$$m_k(\mathbf{p}) = f(x_k) + \mathbf{p}^T \nabla f(x_k) + \frac{1}{2}\mathbf{p}^T B_k \mathbf{p} \tag{3}$$

where $\mathbf{p}$ is the search direction for the current point x_k, $\nabla f(x_k)$ is the gradient, and $\mathbf{B}_k$ is the Hessian or an approximation to it. The gradient of the objective function $\boldsymbol{f}$ can be expressed as:

$$\nabla f(x_k) = \mathbf{J}(x)^{\boldsymbol{T}}\mathbf{r}(x) \tag{4}$$

J is the Jacobian of **r**, containing the first-order partial derivatives of the residual functions:

$$\mathbf{J}(\mathrm{x}) = \begin{bmatrix} \frac{\partial r}{\partial x_1}(x) \\ \vdots \\ \frac{\partial r}{\partial x_n}(x) \end{bmatrix} \quad (5)$$

When the Jacobian cannot be computed analytically, finite differencing to approximate its value is commonly used. In the finite differentiation a step-length (Δ) is assumed. This step-length can be assigned based on the current point (x_i), or as a fixed quantity. The former step-length (Δ1) is defined as a percentage (δ) of the current point as:

$$\Delta 1 = \delta \cdot \mathrm{x}_i \quad (6)$$

In order to avoid the computation of the Jacobian matrix using very small or very large changes in physical parameters, a fixed step-length (Δ2) may be used, defined as:

$$\Delta 2 = \delta \cdot \mathrm{s} \quad (7)$$

where the vector **s** contains scaling factors of δ to increase or decrease the step-length in any variable (x_j). For both cases, the Jacobian in finite differencing is equal to:

$$\mathbf{J}(\mathrm{x}) = \frac{\partial \mathbf{r}}{\partial (x_i)} \cong \frac{\mathbf{r}(x_i+\Delta)-\mathbf{r}(x_i)}{\Delta} \quad (8)$$

The step-length (Δ) can be computed using equation (6) or (7). The success of the gradient-based optimization techniques is strongly dependent on the nature of the multi-dimensional objective function. If the objective function contains local minima, the result of the optimization will depend on the choice of the starting input parameters. To solve these problems, several starting parameters are used.

RESULTS AND DISCUSSION

The procedure of back-analysis for the determination of strength parameters of landslides using FEM and optimization algorithms is illustrated by two examples. For comparative analysis the geometrical models are extracted from the literature. The first corresponds to a homogenous slope inclined 2H:1V (Jiang and Yamagami, 2006) and the second example is a two-layer slope model (Jiang and Yamagami, 2008).

Simple homogenous 2H:1V slope

This example corresponds to a homogenous slope inclined 2H:1V, the soil variables are c=9.8 kPa, ϕ=10º. The soil model is considered to follow the associated flow rule. The factor of safety obtained from FE-SRM is equal to 1.34. Figure 1a shows the finite element model composed of 822 triangular elements (15-noded) with 6741 nodes. The target slip surface (Figure 1b) is determined using the algorithm presented in the previous section. It is based on the strain field computed from a finite element strength reduction analysis. Here, it is considered that the slope failure occurred along the computed slip surface. The task is to change the c-ϕ pair in order to detect this CSS. The objective function is formulated with respect to the observed depths of the slip surface versus the computed numerical locations (Figure 2).

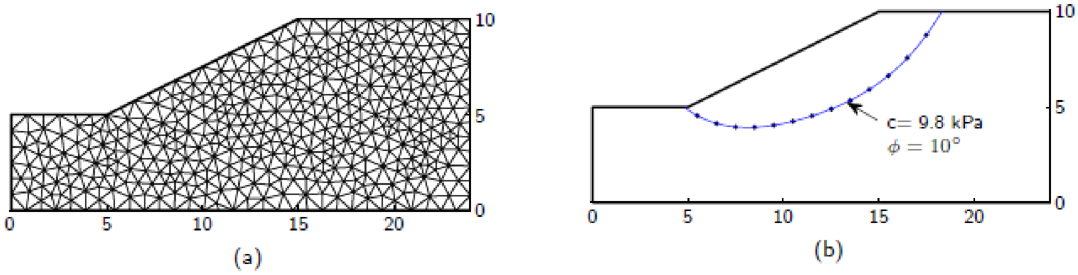
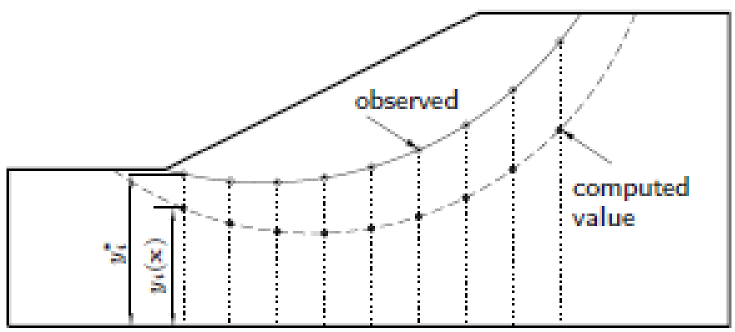


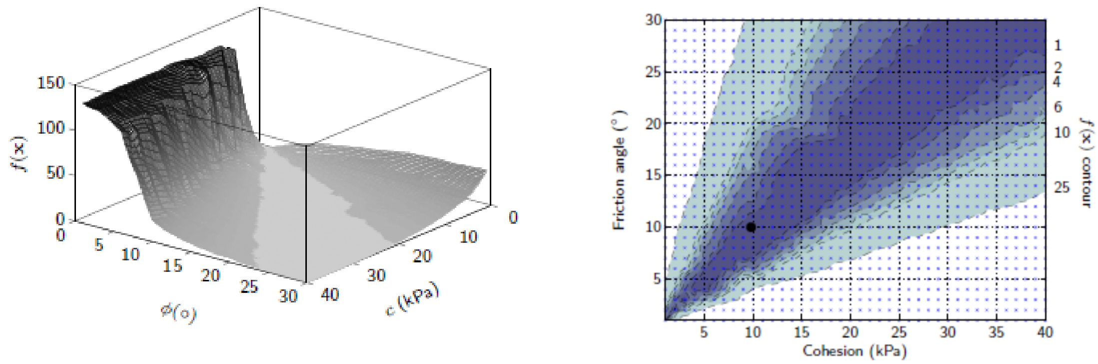
Figure 1: Geometrical configuration for strength parameters back-analysis: (a) mesh discretization; and, (b) target slip surface (•), c = 9.8 kPa, ϕ=10º.



$$\mathbf{r}_i(\mathbf{x}) = y_i^* - y_i(\mathbf{x})$$

Figure 2: Scheme of the determination of the objective function and variables.

The components of the residual vector $\mathbf{r}_i(\mathbf{x})$ contains the discrepancies in the location of the slip surface. The dimension m of residual vector, **r** is 13, and the dimension n of the parameter vector, **x** is 2 (*c* and ϕ). In this example, two parameters are optimized, therefore, it is possible to display the contours of the objective function obtained from changing the cohesion from 1 to 40 kPa and the friction angle ϕ from 1 to 30º in steps of 1 kPa and 1º, respectively. This analysis generates 1200 calculated values from which the contours of the objective function are determined. Figure 3 presents the shape of the objective function with parameters *c* and ϕ. From this surface, the well-known fact that different strength parameter combinations produce the same slip surface is noticed. When the friction angle is low, the discrepancy between the observed and computed slip surface is high. For visualization purposes, the results are presented in a 2D contour plot (Figure 3b). It can be observed that a wide band corresponds to objective function values lower than one (darker area), in this area the difference between the observed and computed slip surface is small $f(\mathbf{x})<1$.



(a) (b)

Figure 3: Contours of the objective function for inverse modeling of failures, a) 3D; and, b) 2D, target c = 9.8 kPa and ϕ=10º (•).

These computed values are used in the inverse modeling where the strength variables are assessed by NLLS optimization algorithm. The NLLS optimization algorithm implemented in MATLAB toolbox is used to back-analyze soil parameters. Since gradient-based algorithms are sensitive to the starting point and the increment used in the computation of the numerical gradient, a parametric analysis is performed. Generally, the increment is defined as a percentage (TolJ) of the trial value ($\mathbf{x}_i$). This increment is used in the finite differencing to approximate the gradient. Depending on the trial value $\mathbf{x}_i$ and TolJ, the sensitivity matrix (gradient) will be computed using values that are very large (e.g., 20º) or very small (e.g., 0.1 kPa). For instance, if the increment is very small, the objective function will be insensitive to this change and no real solution will be found. The influence of the selection of the starting point is analyzed by performing 100 realizations (run for another set of starting values) for different increments (TolJ). When TolJ=5 and 20%, for some trials, the algorithm fails to find the "true" solution. By increasing TolJ to 50% the non-convergence problem, for some cases, is overcome, however, for high starting values (e.g., ϕ=25º), the gradient is computed with huge increments, in this case 12.5º giving a value of 37.5º which goes beyond the limits.

In order to avoid the computation of the sensitivity matrix using very small or very large changes in physical parameters, it is proposed that a fix increment that does not depend on the trial value should be used. This change just affects the gradient algorithm. This modified gradient allows to overcome the problems of very small or very large changes. Moreover, the fix increment is more engineering soundly. For example, by changing 0.1 kPa in the cohesion there will not be any effect in the system. In Figure 4 the results for both the original and modified gradient are shown. It is worth mentioning that the two sets use the same starting values.

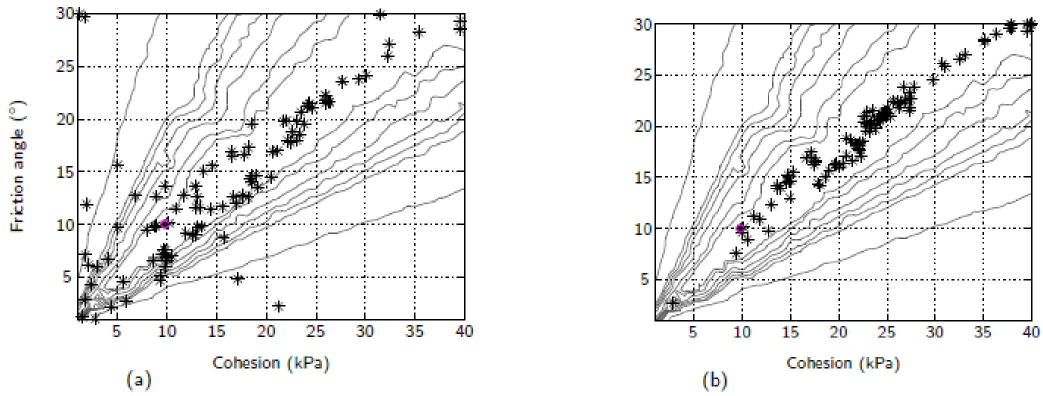


Figure 4: Computed strength parameters (∗) from 100 realizations: (a) original gradient as percentage of trial value; and, (b) modified gradient, increment of 2.

Compared to the original algorithm (Figure 4a), there is an important improvement when the modified gradient (Figure 4b) is used. By using the modified gradient the 100 realizations are inside the lower band, indicating that the algorithm is not trapped in other values. Every result that is inside the lower contour represents an adequate solution of the problem. By dividing the strength parameters obtained from each realization by its factor of safety, the results are presented in normalized form (c/FoS and tanϕ/FoS). Figure 5 shows the spatial distribution of the normalized strength parameters for both original and modified gradient. The normalized actual values are cn=9.8/1.34=7.3 kPa and ϕn= atan(tan10/1.34)=7.47º.

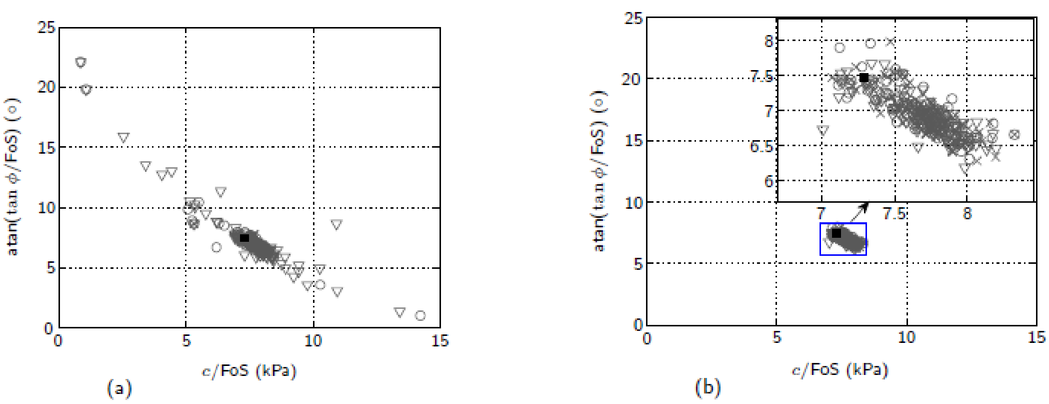


Figure 5: Normalized strength parameters from NLLS, for TolJ equal to 5% (∇), 20% (◦), and 50% (×): (a) original gradient; and, (b) modified gradient.

It can be observed that the points are scattered especially for the original gradient and for TolJ equal to 5 and 20 (see Figure 5b). By using a fix increment, in this case 2º and 2 kPa, the convergence significantly improves, concentrating in a small section.

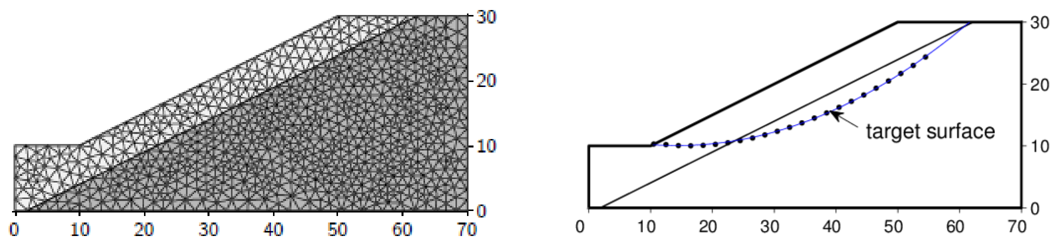
Two layer model

In this section, a two-layer slope model extracted from the literature (Jiang and Yamagami, 2008), is analyzed. The soil parameters used in this analysis for the two strata are listed in Table 1.

Table 1: Increment-size depending on the trial value, **x**i, and TolJ

Soil ID	γ (kN/m^3)	c (kPa)	ϕ (°)
1(top)	17.60	19.60	10
2(bottom)	19.60	4.90	20

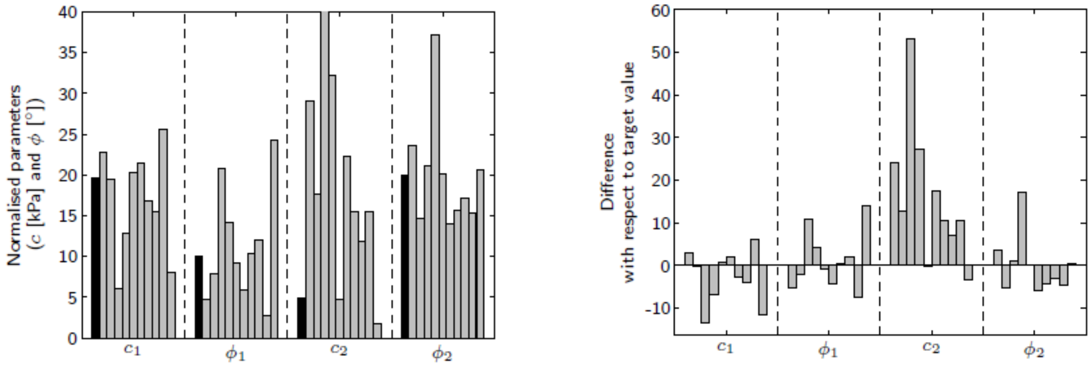
Figure 6a shows the finite element model composed of 1585 triangular elements (6-noded) with 3286 nodes. Additionally, in Figure 6b, the target slip surfaces computed using the finite element method is presented. For the inverse modeling, similar to previous example, the objective function is formulated with respect to the slip surface location (depths FEM). In this case, the dimension m of residual vector is 23 and the dimension n of the parameter vector is 4 (*c1, ϕ1, c2,* and *ϕ2*).



(a) (b)

Figure 6: Geometrical configuration for strength parameters back-analysis of a two-layer slope: (a) mesh discretization; and, (b) target slip surfaces, FEM (•).

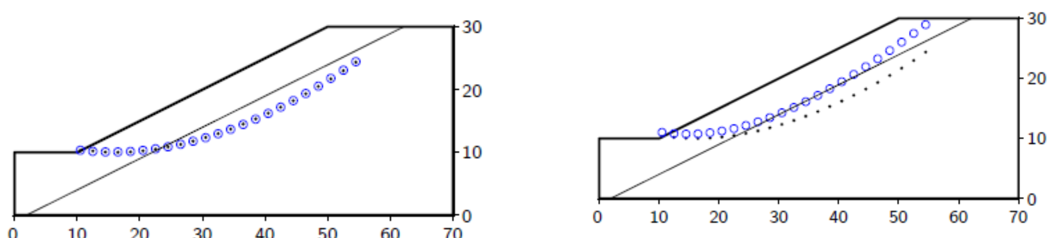
Similar to the single slope example, the optimization algorithm used for this analysis is the NLLS. All the analyses are run using the modified gradient, which provides better convergence. Due to the fact that there are 4 variables, it is not feasible to plot the objective function. For this case, ten realizations are performed. In Figure 7 the soil parameters determined for the ten realizations by the NLLS method are shown. It can be observed that higher differences appear for the cohesion of the second stratum (*c2*) compared to the actual failure parameters.



(a) (b)

Figure 7: Computed parameters for the two-layer slope model by NLLS, target (black bars): (a) normalized parameters with respect to the factor of safety; and, (b) difference with respect to target.

For the other parameters *c1*, *ϕ1*, and *ϕ2*, the higher differences are around 18 kPa and 15° (Figure 7b). The best results are obtained in the fifth realization which gives the value of the objective function *f*(x) =0.024. On the other hand, the worst results are obtained in the fourth realization giving *f*(x) =85.7. The locations of the slip surface for these two cases are presented in Figure 8a and 8b, respectively. For the fifth case, the target and computed slip surfaces coincide, indicating that the NLLS algorithm is able to determine the actual soil parameters (Figure 8a). For the worst case (fourth realization), the computed slip surface is entirely located in the first stratum, differences exist when compared to the target (see Figure 8b).



(a) (b)

Figure 8: Computed slip surface for the two-layer slope model by NLLS optimisation algorithm (◦), target slip surface (•): (a) best result, *f*(x)= 0.024; and, (b) worst result, *f*(x)= 85.7.

CONCLUSIONS

A numerical procedure has been presented combining the FEM and optimization algorithms. The numerical examples show the use of the FEM for the back-analysis of soil strength parameters of slope failures. Two examples acquired from the literature are used as case studies to validate the algorithm. The location of the sliding surface is determined from the shear strain contour. This slip surface is then used as target in the objective function to determine the strength parameters.

It is observed that the sensitivity matrix, for the treated examples produces some problems due to the very small or the large increments that are used depending on the trial value. To overcome this pitfall, a modified gradient has been proposed where a fix step-length, which does not depend on the trial value, is used. This modified gradient used in NLLS computes better results compared with those obtained by the original gradient. This observation is emphasized by the proximity of the computed values to the actual parameters. Using this approach, the full extent of finite element analysis can be exploited by combining the FEM and optimization algorithms. The proposed technique is efficient to determining soil parameters from slope failures by retaining the advantages of finite element slope stability analysis

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