Modal pushover analysis for seismic vulnerability analysis

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Abstract

A simple introduction of inelastic structural analysis for seismic vulnerability by considering the definition of pushover analysis and using a simple elasto-plastic material model to compare analytical and numerical results. The modal pushover method is explained to approximate the response due to "El Centro 1940" ground motion for MDOF systems. The properties of Modal Pushover analysis is used to plot capacity curves vs ground motion spectra in order to perform a one limit state fragility curve using the maximum likelihood method. All explanation are done by following a step by step procedure, then it is applied to a benchmark structure, the 9 storey SAC building.

Introduction

Earthquakes are among the most surprising and aggressive natural phenomena, however now we have more knowledge of their physical origin, and still the vibrations transmitted to the structures remain staggering. From all natural phenomenon only earthquakes produce devastating forces over a relatively small time span, the most recent catastrophic examples such as Kobe (Japan, 1995, M=7.3), Izmit (Turkey, 1999, M=7.6), Boumerdes (Algeria, 2003, M=6.7), Sichuan (China, 2008, M=7.9), Haiti (Haiti, 2010, M=7.0), earthquakes demonstrate the relative weakness of urban environments to such destructive forces.

The number of earthquakes has neither increased nor decreased, approximately 50000 earthquakes occurs on average per year (Dietz 1961), however risk has increased due to the growth of urban population along active seismic faults. The greatest cause of damage in all cases is due to the collapse of masonry buildings often built before the application of construction codes and without control during the construction stage, the latter is a very common practice in third world countries. Despite this, there are countless structures designed without the application of earthquake design rules. These considerations led to the development of seismic vulnerability methods as a methodology to represent the capacity of structures to support seismic ground motions.

This paper is based on a mechanical method using the modal pushover analysis (MPA) (Chopra and Goel 2002) to derive a fragility curve for the probability of reaching the yielding point as a limit state. A step by step procedure details the methodology and results. At the end, the same steps are performed to the 9 storey SAC building, which is used in several papers as a benchmark structure for a non-linear analysis (Goel and Chopra 2004).

The maximum likelihood method (Shinozuka and Feng 2000) is used to plot the fragility curve. The Eurocode 8 type 1 spectrum for different PGA levels is used as target to select 90 scaled ground motions spectrum from the on-line strong-motion record data base "*peer.berkeley.edu*".

Nonlinear analysis

Introduction

An inelastic behaviour of structures on seismic areas should always be considered for in responsible designs. There are several methods based on dynamic and static nonlinear analysis used by engineers to design new buildings. In some cases the analysis of a single building can be performed by a complete dynamic analysis, however for the analysis of existing buildings as is the case for seismic vulnerability analysis, this task can become impossible or hardly difficult to achieve. Therefore nonlinear static analysis is still very common, this method can capture the response parameters that could be obtained by more complex and expensive techniques, e.g., Incremental dynamic analysis (Magliulo et al. 2007).

Pushover analysis

Pushover analysis was introduced in the mid-1970s for a quick evaluation of the vulnerability of naval constructions (Freeman et al. 1975) and for seismic engineering. The pushover analysis is a technique by which a building is subject to monotonically increasing forces with a predetermined unvarying distribution, until a target displacement is reached. The structural resistance is evaluated and the stiffness matrix is updated at every increment of the lateral force, until convergence.

The basic problem is to find the equilibrium of the structure corresponding to the applied loads.

$$\underline{\mathbf{R}}_n - \underline{\mathbf{F}}_n = 0 \qquad (1)$$

Herein, $\underline{\mathbf{R}}_n$ represents the external applied nodal point forces and $\underline{\mathbf{F}}_n$ the corresponding nodal forces from the elements stresses at time t wherein n is a convenient variable to represent the different intensities of load applications at different time steps, this response is carried out by using a step by step incremental solution (Bathe 2006), therefore it considers that the solution at time t is known, and that the solution at the next step $t+\Delta t$ is required.

$$\underline{\mathbf{R}}_{n+1} - \underline{\mathbf{F}}_{n+1} = 0 \qquad (2)$$

$$\underline{\mathbf{F}}_{n+1} = \underline{\mathbf{F}}_n + \underline{\mathbf{F}} \qquad (3)$$

Herein, $\underline{\mathbf{F}}$ is the force corresponding to the element stresses during the increment step. This vector can be approximated using a tangent stiffness matrix ${}^t\mathbf{K}$.

$$\underline{\mathbf{F}} \approx {}^t\mathbf{K}\underline{\mathbf{u}} \qquad (4)$$

Substituting equations 4 and 3 into equation 2 to obtain equation 5

$$ {}^t\mathbf{K}\underline{\mathbf{u}} = \underline{\mathbf{R}}_{n+1} - \underline{\mathbf{F}}_n \qquad (5)$$

The reliability of the method depends on the effective evaluation of $\underline{\mathbf{F}}_n$ and ${}^t\mathbf{K}$.

Pushover curve

An important solution from the pushover analysis is the plot of force vs displacement, usually the bottom shear force vs the top storey displacement. This solution, which comes from a MDOF analysis can be related to the response of an equivalent SDOF system (Krawinkler and Seneviratna 1998) this allows a much simple model for seismic analysis.

Example part I

The model used in this example will be the same through the presentation of all the paper. A 9.144*m* steel column is part of a large viaduct highway system, on the tip a \$766622*kg* mass, which represents the superstructure, see figure 1 and 2. The intention is to find the pushover curve for a continuous element given the following properties.
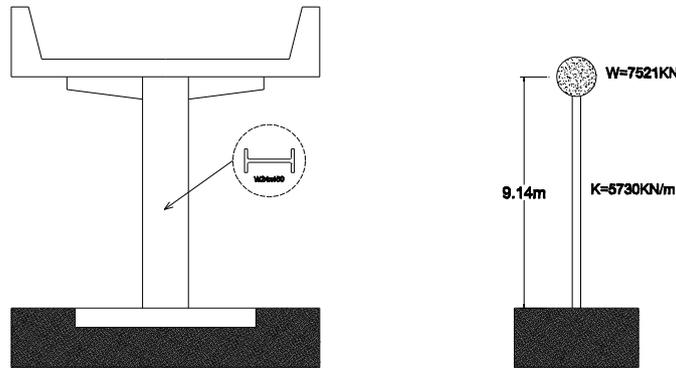
Figure1. Long viaduct

The steel properties are: E=210000*MPa* and v=0.3

The section properties are: $f_y = 235MPa$, $I_{zz} = 7.118*10^{-3}m^4$, $A = 0.0852m^2$
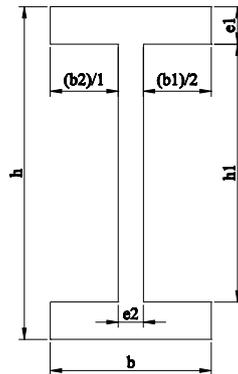
Figure2. W24x450 section

If a constant section is assumed throughout the column height, then:

$$M_y = \sigma_y \left(\frac{bh^3}{12} - \frac{b_1 h_1^3}{12} \right) \frac{2}{h} \tag{6}$$

Equation 6 Represents the yielding capacity moment of the section, and

$$M_u = \sigma_y \left(\frac{bh^2}{4} - \frac{b_1 h_1^2}{4} \right) \frac{2}{h} \tag{7}$$

Equation 7 represents the ultimate capacity moment of the section using an elastic perfectly plastic model. Figure 3 shows the elastoplastic model; figure 4 shows the stress increment. In the latter figure, initially the farthest fibers from the neutral axis reach the yielding stress figure 4a due to M_y, subsequently any moment increment enlarges the yielding area figure 4b and figure 4c, at the end the total section yields figure 4d, which corresponds to M_u.
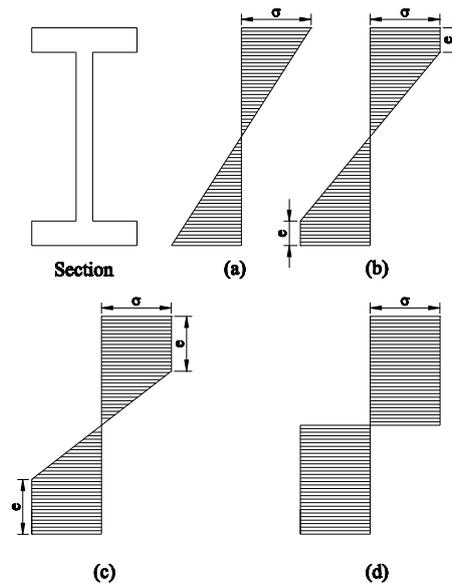
The pushover curve is obtained as a plot of the bottom shear force vs the tip displacement in the X direction; these values correspond to a continuum load increment applied to the top of the column. If V is the bottom shear force and M is the resistant moment, then the first three points are easily obtained for the curve, these are: V=0 for M=0, $V_y=M_y/9.14$ and $V_u=M_u/9.14$; hence there is a linear relation between V and V_y, and a nonlinear relation between V_y and V_u, this nonlinear relation depends on the type of material, section geometry and material law. Here, these points will be obtained by considering a continuous increment of the yielding area.

From figure 2 we can see that $e_1 = (h - h_1)/2$, $e_2 = b - b_1$ and $y_n = h/2 - e_n$; where e represents the increment of the yielding area and it is define for $0 \leq e < h/2$. Assuming that the cross section of the column during bending remains plane and normal to the deflection curve, and that the longitudinal fibers of the column are in condition of simple tension or compression (Timoshenko 1956) then equation 8 represents the radius of curvature for small displacements.

$$1/r = \frac{\sigma_y}{Ey} \qquad (8)$$

Due to the sections geometry (see figure 2) the analysis must be divided in two parts. The first part, considers the yielding area going from e = 0 to e = e_1, the second part when the yielding area goes from e = e_1 to e = h/2.

For the first part $0 \leq e < e_1$

$$M_n = \sigma_y \left(\frac{8by_n^3}{12} - \frac{b_1 h_1^3}{12} \right) \frac{1}{y_n} + \sigma_y \left(\frac{b\,h^2}{4} - by_n^2 \right) \qquad (9)$$

For the second part $0 \leq e < h/2$

$$M_n = \sigma_y \left[\frac{2}{3} e_2 y_n^2 + be_1(h_1 + e_1) + e_2(e_n - e_1)(2y_n + e_n - e_1) \right] \qquad (10)$$

This methodology consists of increasing e in order to calculate y, r and M. For the section in figure 2, M_y = 4535.50kN-m and M_u=5459.38kN-m; M_u is the value when all the area has yield (see figure 4 part d), $M_u = \sigma_y \left(\frac{bh^2}{4} - \frac{b_1 h_1^2}{4} \right)$ When e_n=h/2, y_n=0 the displacement in equations 8 and 11 tend to infinity and the moment tends to M_u; see figure 5.

In order to obtain force *vs* displacement for the pushover curve is necessary to define a relation for the top deformation and y_n. This is done by using the stiffness ($K = 3EI/L^3$) of a cantilever beam with a tip load to then obtain equation 11, which allows plotting figure 6.

$$\Delta = \frac{\sigma_y L^2}{3E y_n} \tag{11}$$
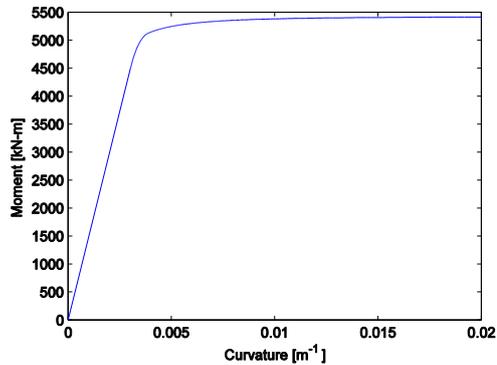
Figure 5. Moment of curvature
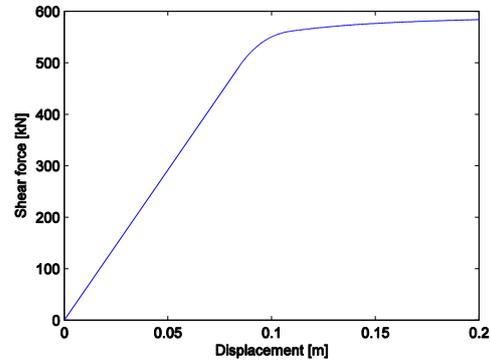
Figure 6. Pushover curve

This example showed a simple incremental method to derive a pushover curve for a single element, considering a tip load and using an elastoplastic material model.

Modal pushover analysis (MPA)

In the Modal pushover analysis the system is converted into a MDOFs divided in m elements, then the load is applied in the nodes with a distribution corresponding to the principal mode shapes of the structure; therefore different pushover curves are obtained for each load configuration.

$$\underline{s}_i = \mathbf{LM}\underline{\phi}_i \tag{12}$$

Equation 12 represents the shape of the forces to be applied to the structure, M and ϕ are the mass and mode shape matrices, L is the influence diagonal matrix which indicates the excited degrees of freedom. To achieve good results, it is recommended that an effective modal mass of at least 90% of the total mass of the structure should be considered in the analysis (Gueguen 2013), in the majority of cases this requirement is achieve with the first three modes.

Example part II

The same example is modelled now using 16 elements, and then the eigenvalue problem is solved for a system with 48 degrees of freedom for which a plot of the first three modes shapes is obtained. The computations are performed using STABIL2.0 toolbox (Jansen 2010), the modes and principal frequencies are shown in figure 7.

Matrix L from equation 12 is defined in such way that only the horizontal degrees of freedom are excited. Figure 8 illustrates the distribution of horizontal forces corresponding to each of the first three mode shapes using equation 12.

$$\Gamma_i = \frac{\phi_i^T M \iota}{\phi^T M \phi} \tag{13}$$

$$M^* = \Gamma_i^2 \tag{14}$$

Mode 1 0.44Hz   Mode 2 43.65Hz   Mode 3 139.26Hz
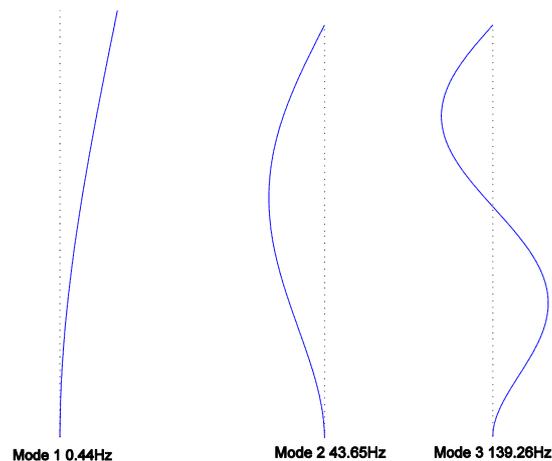
Figure 7. Principal mode shapes

Equation 13 represents the modal participation factor; equation 14 represents the mass participation factor. The total mass including self-weight is 772735.04kg, then from equation 14, the corresponding mass participation M^* for mode 1 is 769761.33, mode 2 1579.96 and mode3 484.24. If M^* is compared with the total mass, then the mass participation factor for mode 1 is 99.62%, for mode 2 0.2% and for mode 3 0.063%. Thus it is only necessary to consider the first mode to obtain an accurate solution.

Example part III, comparing results with SAP2000

The same 16 elements model is implemented in SAP2000; an elastoplastic no hardening material model is considered. The horizontal loads obtained from equation 12 are implemented as initial loads to start the pushover analysis.

Figure 9 shows the plot of the analytical and numerical solution of the pushover curve, the correspondence is almost perfect; the small difference can be due to convergence of SAP2000 near to the yielding point.

This solution is important because the response of a MDOF can be related to the response of a combination of SDOFs. This can be extended to quantify the displacement of SDOF systems using spectra analysis. However in this example this result was easily obtained due to the dominance of the first mode, when more than one mode is important, then a relationship between modes and pushover curves is required.
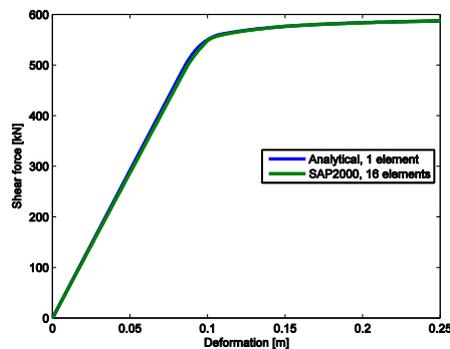
Figure 9. Analytical and numerical solution

Relation between spectra analysis and pushover analysis

If the displacement imposed from a ground motion to a SDOF is known, then it could be used to define the target displacement for the pushover analysis and find the structural demands. This can be done by plotting the corresponding earthquake spectra versus the capacity curve; the latter is obtained by transforming the pushover curve in a plot of spectra acceleration in function of spectra displacements.

$$\mathbf{M}\underline{\ddot{u}} + \mathbf{C}\underline{\dot{u}} + \underline{\boldsymbol{f}_s}(u, \dot{u}) = \mathbf{M}\underline{\iota}\ddot{u}_g(t) \qquad (15)$$

$$\underline{\mathbf{u}}(t) = \boldsymbol{\phi}\underline{\alpha}(t) \qquad (16)$$

If

$$u_{ij}(t) \approx \underline{\phi}_i \alpha_j(t) \qquad (17)$$

Where

$$\Gamma_j = \underline{\phi}_j M_{ij} \iota_i \qquad (18)$$

$$\ddot{D} + 2\xi_i \omega_i \dot{D} + \frac{F_i}{\Gamma_i} = -\ddot{u}_g(t) \qquad (19)$$

$$\alpha_i = \Gamma_i D(t) \qquad (20)$$

The resistant force at any stage of the nonlinear static procedure:

$$\underline{\boldsymbol{f}}_s = \kappa \mathbf{M}\underline{\phi}_i \qquad (21)$$

The base shear force at any stage of the nonlinear static procedure:

$$V = \underline{\iota}^T \underline{\boldsymbol{f}}_s \qquad (22)$$

$$\kappa = \frac{V}{\Gamma_i} \qquad (23)$$

In case the case of elastic analysis equation 22 can be written as: $V = \underline{\iota}^T \mathbf{K}\underline{\mathbf{u}}$, which can be simplified to obtain:

$$S_a = \frac{F_{si}}{\Gamma_i^2} \quad \text{And} \quad D_n = \frac{u_{rn}}{\Gamma_i \phi_{rn}} \qquad (24)$$

Equation 24 can be extended to use it for nonlinear analysis. Equation 19 details an approximate solution for inelastic analysis, wherein a MDOFs is replaced by the combination of SDOF systems, *i* goes from 1 to the number of representative modes; for each mode, a pushover curve is obtained.

Equation 24 is use to relate the pushover force in spectra coordinates, and it is called the capacity curve. This representation can be approximated by a bilinear function using the equivalent area method.

In the equivalent area method used, first the initial and last point (target point) are fixed; then the original stiffness is used to draw the first part of the curve starting from the origin to an equivalent yielding point D_y which then is fixed by balancing the area of the original curve with the area of the bilinear curve.

Equation 24 is used to transform the pushover curve into the capacity curve; herein the ordinates are represented by spectra acceleration and the abscissa by spectra displacement of the equivalent SDOF system. This important deduction makes possible to compare capacity curve with earthquake spectra

plot (figure 12), their intersection in the elastic range represent the displacement demand. Several ground motions can be compared with the same capacity curve, which is very helpful for the analysis of seismic vulnerability.
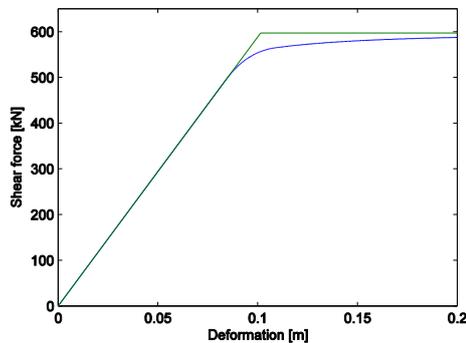
Figure 10. Bilinear approximation
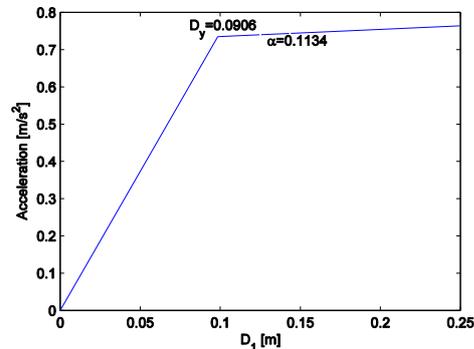
Figure 11. Capacity curve
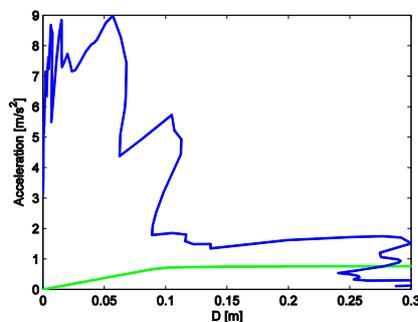
Figure 12. "El Centro" spectrum & mode 1 capacity curve
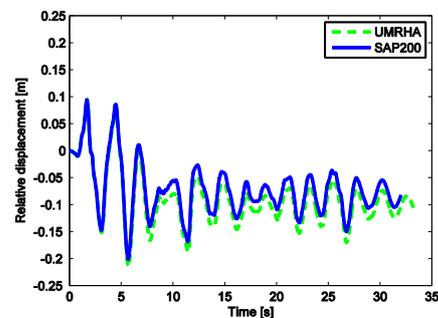
Figure 13. Tip displacement

Approximate nonlinear dynamic solution

Dynamic analysis can be performed using an approximate solution, where the uncoupling of modes is extended to inelastic systems using equation 19; this equation represents the dynamic differential equation of a SDOF system corresponding to mode *n*, then equation 16 and equation 18 are used to return to normal coordinates. The inelastic behaviour is represented by the tangent stiffness using the bilinear approximation obtained from the modal pushover analysis. Each response can be combined in the same manner as in the case of elastic analysis. An explicit or implicit time integration method can be used. Figure 13 compares the tip displacement due to ``El Centro'' ground motion by considering the approximate and the full transient solution, in both cases using the Newmark method.

Fragility analysis

Introduction

A statistical procedure for fitting fragility functions to structural analysis data when the structural analysis is performed using different ground motions at each intensity level. A well accepted methodology for such cases is the maximum likelihood estimation method (MLE) (Shinozuka et al. 2000) which is also is easy to implement.

MLE can be implemented for a variety of situations; however this paper will be focus on collapse fragility functions obtained from structural analysis data. The structural response quantity to estimate

from structural analyses is the probability of collapse (exceed the elastic capacity) as a function of ground motion intensity using spectral acceleration.

Conditional spectrum

A conditional type 1 spectra from EuroCode 8 is generated as target spectrum, it will be used to select different ground motions at each PGA level. For each level, a group of 90 ground motions are selected using the *peer.berkeley.edu/* PEER Ground Motion Database.

A type a soil and 0.05 damping ratio are used to build the spectrum. The PGA level varies from 0.1g to 1g, with an increment of 0.05g and two additional values of 1.3g and 2g. Figure 14 shows the different ground motion spectra used to fit a 0.25g spectrum.
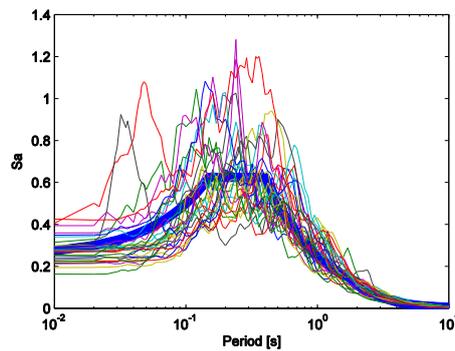
Figre 14. Target spectrum for PGA = 0.25g

Maximum likelihood method (MLE)

The MLE method is used to account for the non-constant variance of the observed fractions of collapse (Shinozuka et al. 2000). First, the probability of observing z_j collapses out of n_j ground motions with Sa=x_j is given by the binomial distribution.

$$P(z_j/n_j) = \binom{n_j}{z_j} p_j^{z_j}(1-p)^{n_j-z_j} \qquad (25)$$

Where P_j is the probability that a ground motion with Sa = x_j will exceed the yielding point of the structure, which can also be denoted as P(C\Sa = x). The goal is to find the fragility function which best fits *p*, the MLE estimation chooses the function with the highest probability of observing the limit state considering all Sa values. This is done by taking the product of binomial probabilities at each PGA levels.

$$\text{Likelihood} = \prod_{j=1}^{m} \binom{n_j}{z_j} p_j^{z_j}(1-p)^{n_j-z_j} \qquad (26)$$

Herein *m* represents the number of Sa values for which *z* and *n* are already known; the fragility function will be the one that maximizes this likelihood. A log normal cumulative distribution function is used to fit the data.

$$P(C|S_a = x) = \Phi\left(\frac{lnx - \mu}{\beta}\right) \qquad (27)$$

Replacing equation 27 into equation 26 to obtain:

$$\text{Likelihood} = \prod_{j=1}^{m} \binom{n_j}{z_j} \Phi\left(\frac{lnx_j - \mu}{\beta}\right)^{z_j}\left(1 - \Phi\left(\frac{lnx_j - \mu}{\beta}\right)\right)^{n_j-z_j} \qquad (28)$$

To obtain the fragility function parameters, it is necessary to maximize the likelihood function.

$$\hat{\mu}, \hat{\beta} = \frac{max}{\mu, \sigma} \prod_{j=1}^{m} \binom{n_j}{z_j} \Phi\left(\frac{lnx_j - \mu}{\beta}\right)^{z_j} \left(1 - \Phi\left(\frac{lnx_j - \mu}{\beta}\right)\right)^{n_j - z_j} \tag{29}$$

The values for the right side of equation 28 are known from the pushover analysis. The Matlab Statistical toolbox is used to perform the optimization and find $\hat{\mu}$ and $\hat{\beta}$.

Example part IV, fragility curve

A comparison between the modal pushover analysis curve and each of the spectra for a given PGA value, allows determining an intersection, which will exceed or not the yielding point. If the intersection exceeds, then *1* is assigned to a variable, and if it is not exceeded *0* is assigned to the variable, this is implemented in a subroutine using Matlab. In figure 15, three spectra and the capacity curves are plotted for PGA = 0.25g; two of the spectra intersect the capacity curve on the left side of the yielding point and one at the right side, hence the variable would storage (0 0 1) which can resume the procedure used to count observations; this is repeated for all PGA levels.
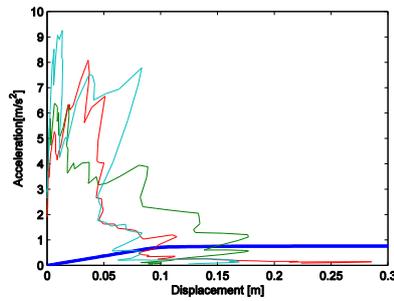
Figure 15. Spectra and capacity curve   **Figure 16.** Yielding point fragility curve
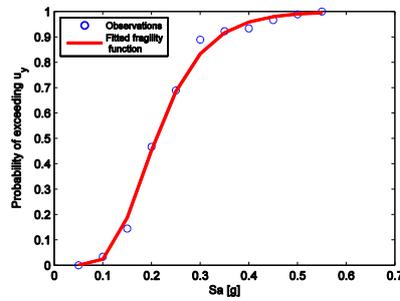
Table 1. Collapses

PGA	Collapses
0.10	3
0.15	12
0.2	42
0.25	62
0.3	80
0.35	83
0.4	84
0.45	87
0.5	89
0.55	90

For the cantilever structure, PGA values above 0.55g produce 90/90 collapses, and then it is not necessary to calculate for higher levels. The information data of the number of collapses, equation 29 and Matlab Statistical toolbox are used to plot the fragility curve, which is shown in figure 16.

Sac 9 storey building

Herein the procedures explained in detail in the previous sections will be applied to a benchmark structure, the SAC 9 storey building. The resume of characteristics for this structure is presented next.

Steel structure: Columns 345MPa, beams 248MPa

Table 2. Sections

Beams	Level	Columns	Level
W36x160	Ground – 2nd level	W14x500	Basement – 1st level
W36x135	3rd – 6th level	W14x455	1st – 3rd
W30x99	7th level	W14x370	3rd – 5th level
W27x84	8th level	W14x283	5th – 7th level
W24x68	9th level	W14x257	7th – 9th level

Geometry

Bay widths: 9.15m.

Storey heights: base level 3.65m, ground level 5.49m, 1st-8th 3.96m

Modelling: 3 internal nodes for beams and 2 for columns. In total 337 nodes and 380 elements.

Boundary conditions: Nodes 1- 6 fixed; nodes 19 and 24 $u_x = 0$.
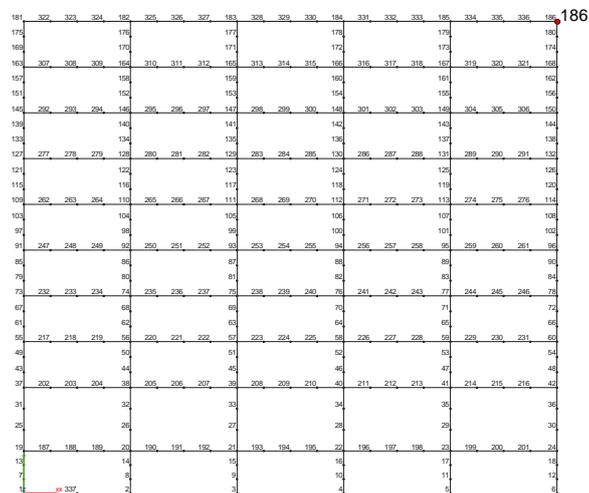
Figure 17. SAC 9 storey building

Modes and natural frequencies

The model is created using SAP2000 and also using Stabil2 Toolbox. The first three modes and natural frequencies are shown in figure 18.

Mass participation factors

The number of modes to be included will have to capture at least the 90% of the total mass. The first mode contributes with 76%, the second with 10% and the third with a 4%. Hence, the first three modes will be used for the analysis.

Modal pushover forces

From equation 12 the forces are obtained and applied to the left lateral nodes of the structure considering only horizontal degrees of freedom.
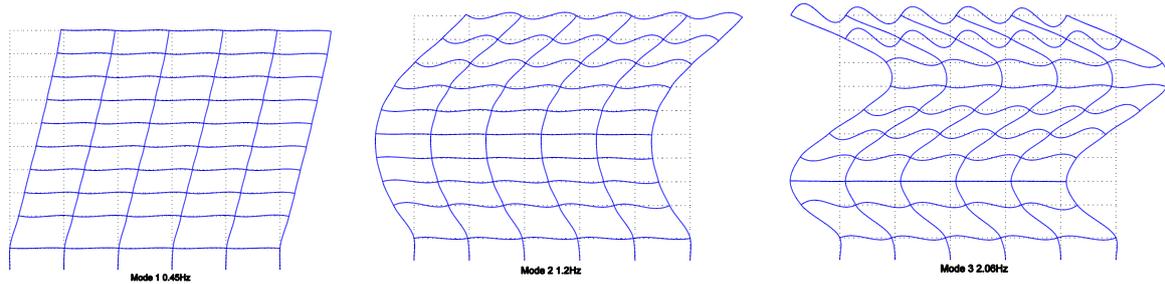
Figure 18. Modes and natural frequencies

Figure 19 shows the lateral forces for the first three modes. These forces are applied to the structure and analysed using SAP2000, thereafter three pushover curves are obtained. The three pushover curves and their bilinear approximation are shown figure 21.

Bilinear capacity curves

The bilinear capacity curves are obtained from the bilinear pushover curves using equation 24, see figure 22.
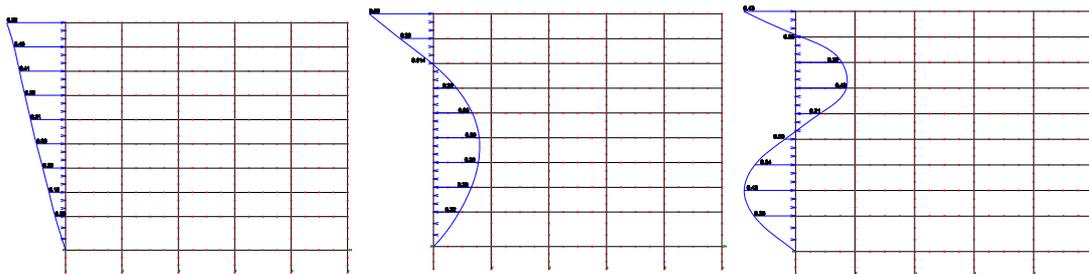
Figure 19. Pushover force mode1,2 and 3
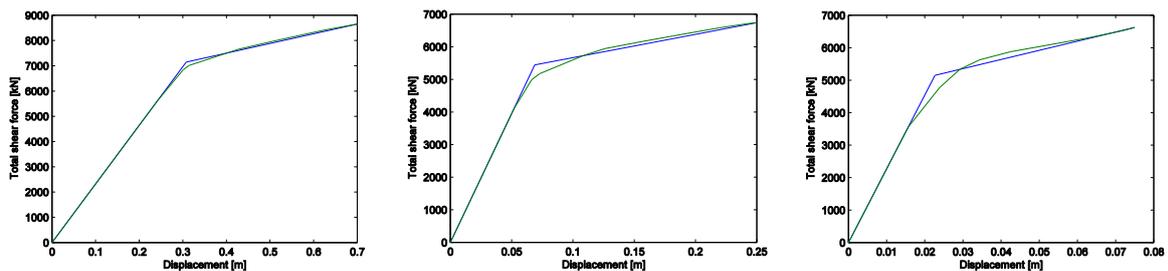
Figure 21. Pushover curves
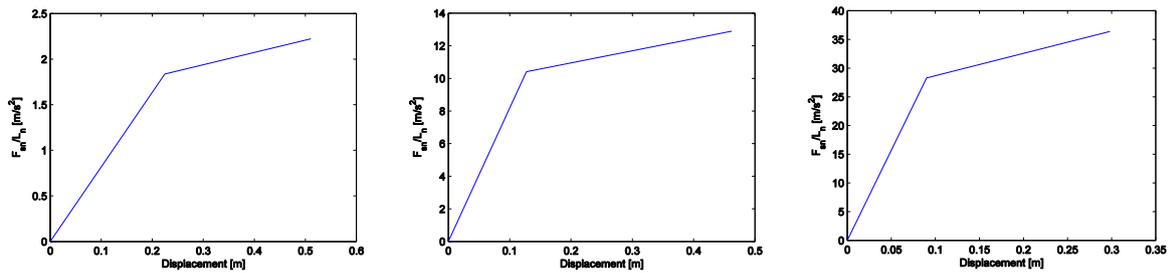
Figure 22. Capacity curves

Dynamic analysis

If each of the three capacity curves from figure 21 is used to solve their corresponding nonlinear SDOF, then from equations 20 and equation 17 the approximate responses can obtain as results for the MDOFs. Figure 23 shows the solution for the "El Centro" earthquake, which does not exceed the yielding point of the structure.

MPA and uncoupled modal response history analysis

The Modal Pushover Analysis and the Uncoupled Modal Response History Analysis (UMRHA) are compared using the "exact" transient solution.

First, it is necessary to clarify the dependence between both methods, the UMRHA uses the modal pushover curves as material models to solve equivalent SDOF systems, then the maximum displacement obtained from each of the UMRHA are used as the target displacement in the MPA to obtain the elements responses. In order to compare their accuracy, the response of lateral floor displacements is obtained and plot in figure 25. It is important to point out that the MPA displacement target can always be found using the UMRHA.
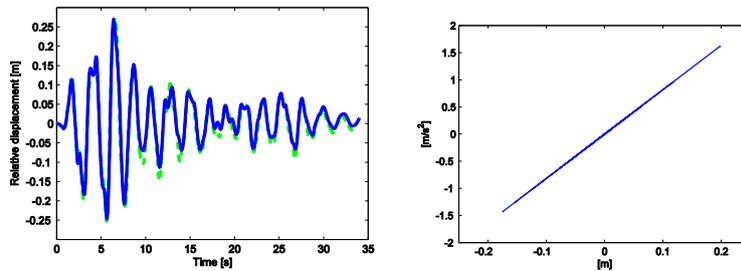
Figure 23. Node 186 displacement and hysteresis for the "El Centro" ground motion
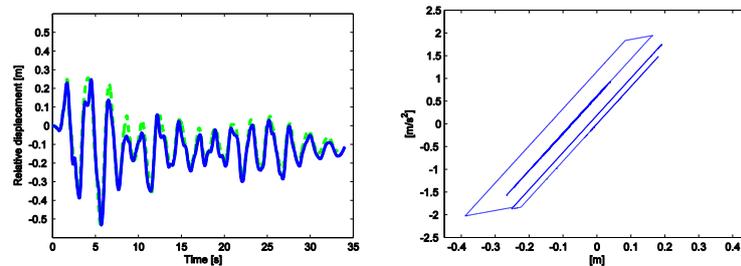
Figure 24. Node 186 displacement and hysteresis for the "El Centro" ground motion x2

Figure 25 illustrates five curves, the vertical curve represents the transient solution, three curves represent the pushover solution considering one mode, two modes and three modes respectively, the forth curve represents the approximate dynamic solution. The modes were combined using the SRSS rule. From figure 25 it is clear that both methodologies are accurate to predict the top storey displacement (node 186), and that smaller errors are obtained using the MPA. This can be justified considering that the modal push over analysis is a real non-linear analysis, due to the stiffness matrix, which is updated at each increment and no simplifications are used in the material model. The UMRHA instead is based in some approximations, first the coupling of modes is neglected, and secondly the pushover curve is simplified by a bilinear function.
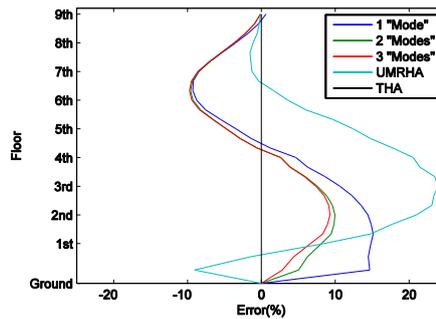
Figure 25. Hise-wise variation of error in floor displacement for 2 x "El Centro"

In the case of elastic analysis it can be simply obtained from the capacity curve, however for inelastic structures, the energy dissipated during the plastic deformation changes the system behaviour and the capacity curves is not always accurate to predict an approximate solution (Chopra and Goel 1999) Figure 24 shows the response using the "exact" solution and approximate solution for node 186 imposed by the ``El Centro" ground motion scaled by a factor of two, now the yielding is evident in the structure.

Fragility analysis

The same procedure applied to the cantilever column will be used to plot the fragility curve for the SAC structure. Figure 26 shows the fragility curve for the SAC building.

Table 3. Counting collapses

PGA	Collapses	PGA	Collapses
0.10	0	0.55	63
0.15	0	0.60	66
0.20	1	0.65	67
0.25	2	0.70	69
0.30	5	0.80	71
0.35	14	0.90	72
0.40	23	1.00	74
0.45	29	1.30	77
0.50	50	2	85
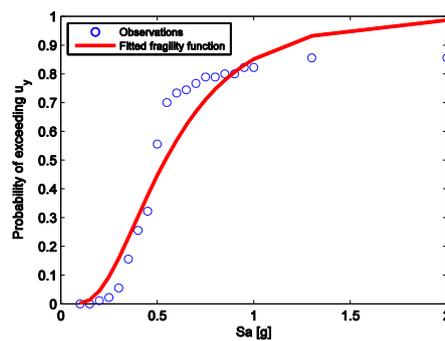
Figure 26. Fragility curve SAC building

If the fragility curve from figure 26 is compared with the fragility curve for the cantilever column, then the former is a much safer structure, due to a 50% probability of collapse for an PGA=0.5g level versus the 50% possibility of collapse but for an PGA=0.2g level.

Conclusions

A benchmark SAC 9 storey building has been chosen, starting from a dynamic analysis where the three principal modes were found as representative of the structure; then the MPA was applied to find the bilinear approximations for the pushover and capacity curves, the latter were used as material models for the UMRHA to obtain the relative displacements on node 186. The maximum displacement from the UMRHA method was used to recalculate the pushover curve considering the exact target. A comparison between MPA and UMRHA using the "exact" transient solution for floor displacement showed a smaller error for the MPA analysis. The fragility analysis using structural data and the Likelihood method for a single limit state derived the fragility curve.

An important conclusion obtained from the analysis of the capacity curves and seismic demand is the fast attainment of results, which is useful for vulnerability assessment of structures; however the difficulty to know the target displacement for inelastic structures remains. A generalization of the capacity curve and seismic demand (spectra demand) analysis can be used to overcome this difficulty, where the seismic demand is properly reduced by an over damped or inelastic approach. These methods are recommended by ATC-40, Euro code 8, HAZUS, FEMA-356 and other technical codes.

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