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Dissipative output feedback control for semi-Markovian jump systems under hybrid cyber-attacks

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Abstract

In this paper, the dissipativity-based dynamic output feedback controller (DOFC) design for Semi-Markovian jump systems under stochastic cyber-attacks is first proposed. It is assumed that the time-varying uncertainties obey Bernoulli-distribution and transition probability matrix is time-varying and partially accessed. By utilizing the dissipativity-based technique, sufficient conditions for the existence of the DOFC are obtained to ensure the exponential stability with a strict dissipative performance of the resulted system. Next, the proposed results are improved by fractionalizing the time-varying transition probability matrix and the corresponding DOFC gains are obtained by cone complementarity linearization algorithm. Simulations results are provided to demonstrate the effectiveness and theoretical value of the proposed dissipativity-based DOFC design method.

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1. Introduction

Markovian jump systems (MJSs) have been attracting a great deal of attention ranging from theoretical research to practical applications, due to their powerful modelling capabilities to handle inevitably sudden changes caused by abrupt environment disturbances, random failures or component damages, or even human factors in normal operation [1,2]. Over the past few decades, enormous practical applications of MJSs have appeared in economics [3], computer and communication systems, solar thermal receivers, flight systems, power systems [4], etc. Considerable attention has also been paid to the various aspects of analysis and synthesis of MJSs. For example, recent developments of modelling, filter design, stability analysis and sliding model control for MJSs were summarized in [5–7]. Robust $\mathcal{H}_\infty$ control for discrete-Time MJSs was discussed in [8] and the related results were improved for robust $\mathcal{H}_\infty$ filtering of continuous-time nonhomogeneous MJSs in [9].

However, MJSs show limitations in practical applications by assuming that the sojourn-time follows an exponential probability distribution and the corresponding transition probabilities (TPs) are constant, which may unfortunately cause conservativeness for the memoryless property [2]. To mitigate this problem, Semi-Markovian jump systems (Semi-MJSs) come to being by assuming that the sojourn-time follows other probability distribution rather than an exponential distribution, and that the TPs are time-varying. For example, it is hard and costly to estimate or measure all the accurate values of TPs between different nodes across the Internet; in network communication system, the network-induced transition rate are generally time-dependent for the time-varying transition resources [10]. From this point of view, the traditional MJSs can be considered as a particular class of Semi-MJSs, which motivates researchers to work on analysis and synthesis of Semi-MJSs with different transition rate matrices (TRMs), such as completely known TPM, partially unknown TPM [11,12], uncertain TPM [13,14] and polytopic time-varying TPM [15]. As an example of this line of research, [16] investigated the system stability analysis and $\mathcal{H}_\infty$ control synthesis problems for a special class of continuous-time MJSs based on input-output approach, which includes exactly known, uncertain and partially unknown TPs simultaneously.

Until now, various control or filtering schemes of Semi-MJSs have been investigated. The $\mathcal{H}_\infty$ control for continuous-time MJSs has been considered in [16]. In this paper, a memory state feedback controller was designed based on the input-output approach. One of the advantages of this control method was the strong robustness to disturbance and uncertainties. The adaptive neural output-feedback fault-tolerant control of switched nonlinear systems with unmodelled dynamics by using small-gain technique was studied in [17]. The results in [18,19] considered the asynchronous passive/dissipative control problem for discrete-time MJSs, where the states were measurable. Besides, there are also many controller design schemes for the system with unmeasurable states[20]. The static output feedback control for continuous-time Semi-MJSs based on the linear matrix inequalities (LMI) approach was investigated in [21], and the dynamic output feedback controller (DOFC) design problem for MJSs was studied in [22–26]. Besides, techniques of stochastic systems have been adopted to handle with the security control problem of networked control systems under cyber attacks. For example, the event-triggered DOFC problem for discrete-time MJSs under deception attacks was investigated in [27], and the observer-based OFC problem for a class of cyber-physical systems under periodic denial-of-service (DoS) attacks was proposed using switching strategy in [28]. Furthermore, the event-triggered $\mathcal{H}_\infty$ load frequency control problem for multiarea power systems under hybrid cyber attacks was conducted in [29] using event-triggered scheme.

However, the dissipativity-based DOFC problem for continuous Semi-MJSs under hybrid cyber attacks is unsolved.

Furthermore, the dissipativity theory was first brought in by Willems in [30] and has been a more general and popular property in analysis and synthesis of MJSs for the input-output energy-related performance [31,32]. Dissipativity, which is described by the supply rate and input/output energy functions, comes from electrical networks and arouses that the related dissipativity systems have a critical performance that the dissipativity systems can not generate energy but only can dissipate energy. That is, the total energy stored in dissipativity systems decreases [33]. Motivated by the above consideration, the main objective of this paper is to design the DOFC problem for a special class of continuous-time Semi-MJSs under hybrid cyber attacks, by fully taking advantages of the dissipativity-based techniques. Compared with existed results, the main contributions of this paper are summarized as following. Given a continuous-time uncertain Semi-MJS with partially unknown time-varying TPM in Eq. (1), based on the dissipativity-based technique and parameter-dependent Lyapunov function, we

(1) consider hybrid cyber attacks, including DoS attacks and deception attacks;
(2) develop sufficient conditions for the existence of the DOFC, ensuring that the resulting system in Eq. (7) is exponentially mean-square stable with strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative;
(3) improve the proposed results by fractionalizing the time-varying TPM with partially unknown elements; and
(4) solve the dissipativity-based DOFC design problem based on the cone complementarity linearization algorithm.

The reminder of this paper is given as follows. The system description and problem formulation including some basic concepts and lemmas are given in Section 2. Main results on stability analysis and DOFC design for uncertain Semi-MJSs with hybrid cyber attacks are presented in Section 3. In Section 4, a numerical example is conducted to demonstrate the effectiveness of the proposed DOFC design method.

Notations. The following notations are used throughout this paper. The superscripts -1 and T stand for the matrix inverse and transposition; notations $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ are the largest and smallest eigenvalues of a matrix, respectively. In addition, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ represent the Euclidean space of n-dimension and the set of all real matrices of $(n\times m)$-dimension, respectively. Moreover, the space of square integrable functions over $[0, \infty]$ with the norm $\|\cdot\|_2$ is denoted by $\mathcal{L}_2[0, \infty)$, and the mathematical expectation is denoted by $\mathbf{E}[\cdot]$. The notation sym$\{A\}$ is a shorthand for $A + A^T$ and diag$\{\cdot\}$ denotes a block diagonal matrix. In symmetric block matrices, an asterisk $*$ is used to denote a term that is induced by symmetry.

2. System description and preliminaries

In this section, we will focus on some basic concepts on the system description and problem statement, which are essential to the derivation of the main results.

2.1. System description

Consider the following continuous-time stochastic systems with Semi-Markov parameters and randomly occurring uncertainties (ROUs) over a probability space $(\Omega, \mathcal{F}, P_r)$:

$$\begin{cases} \dot{x}(t) = A(\eta_t, t)x(t) + B_1(\eta_t)\omega(t) + B_2(\eta_t)\bar{u}(t), \\ z(t) = C_1(\eta_t)x(t) + D_{11}(\eta_t)\omega(t) + D_{12}(\eta_t)\bar{u}(t), \\ y(t) = C_2(\eta_t)x(t) + D_{21}(\eta_t)\omega(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $z(t) \in \mathbb{R}^p$ and $y(t) \in \mathbb{R}^q$ are the state vector, controlled output, and measured output, respectively; $\bar{u}(t) \in \mathbb{R}^m$ is the control input; $\omega(t) \in \mathbb{R}^l$ is the exogenous disturbance input which belongs to $\mathcal{L}_2[0, \infty)$. Let $\{\eta_t, t \geq 0\}$ be a Semi-Markov chain taking values in the state space $\mathcal{N} = \{1, 2, \ldots, N\}$ and assuming that the time-varying TPs satisfy:

$$\Pr(\eta_{t+h} = j|\eta_t = i) = \begin{cases} \alpha_{ij}(h)h + o(h), & \text{if } i \neq j, \\ 1 + \alpha_{ij}(h)h + o(h), & \text{if } i = j, \end{cases}$$

where h is called the sojourn time and $o(h)$ is the little-o notation, satisfying $h > 0$ and $\lim_{h \to 0} \frac{o(h)}{h} = 0$; $\alpha_{ij}(h)$ is the sojourn-time-based TP rate (TPR) from subsystem i at time t to subsystem j at time $t + h$, and satisfies $\alpha_{ij}(h) \geq 0 (i, j \in \mathcal{N}, i \neq j)$ and $\alpha_{ii}(h) = -\sum_{j=1, j \neq i}^{N} \alpha_{ij}(h)$. Motivated by [15], we assume the time-varying TRM $\Pr(t) \triangleq (\alpha_{ij}(h))$ to be a polytope with vertices $\Pr^r (r = 1, 2, \ldots, S)$, that is

$$\Pr(t) = \sum_{r=1}^{S} k_r(t) \Pr^r, \ \sum_{r=1}^{S} k_r(t) = 1, \ k_r(t) \geq 0,$$

where $\Pr^r$ $(r = 1, 2, \ldots, S)$ refers to the vertices of the polytope and S is the number of vectors of $\Pr^r$; $k_r(t)$ is a parameter vector with bounded changing rate, that is,

$$-\delta_r \leq \dot{k}_r(t) \leq \delta_r, \quad \delta_r \geq 0, \quad r = 1, 2, \ldots, S - 1.$$

Furthermore, we assume the time-varying TPs to be partially unknown, that is, some elements of the TPM are uncertain and others are completely unknown. For example, for Semi-MJS in Eq. (1) with four subsystems, the TPM $\Pr^r$ can be expressed as

$$\Pr^r = \begin{bmatrix} \alpha_{11}^r & \alpha_{12}^r & ? & ? \\ ? & ? & \alpha_{23}^r & \alpha_{24}^r \\ \alpha_{31}^r & ? & \alpha_{33}^r & ? \\ \alpha_{41}^r & ? & ? & ? \end{bmatrix},$$

where "?" denotes the inaccessible elements. For notational clarity, we define $\mathcal{N} = \mathcal{N}_{uk}^i \cup \mathcal{N}_{uc}^i (i \in \mathcal{N})$ and

$$\mathcal{N}_{uk}^i \triangleq \{j : \alpha_{ij}^r \text{ is unknown}\}, \ \mathcal{N}_{uc}^i \triangleq \{j : \alpha_{ij}^r \text{ is uncertain}\}. \tag{2}$$

Remark 1. Note that the time-varying TPs are more natural to MJS, covering the known, uncertain and completely unknown elements. The proposed DOFC design method is less conservative in reducing the time-cost and complexity to estimate or measure all the accurate TPs [16].

Furthermore, matrices $A(\eta_t, t)$, $B_1(\eta_t)$ and $B_2(\eta_t)$ are state matrices with ROUs and appropriate dimensions, that is,

$$A(\eta_t, t) = A(\eta_t) + \beta_1(t) \Delta A(\eta_t, t), \quad \Delta A(\eta_t, t) = M(\eta_t) F(\eta_t, t) N(\eta_t).$$

Matrices $A(\eta_t)$, $B_1(\eta_t)$ and $B_2(\eta_t)$ are known real matrices, matrices $M(\eta_t)$ and $N(\eta_t)$ denote the known matrices of the time-varying uncertainties, and the unknown matrix $F(\eta_t, t)$ stands for the time-varying part $\Delta A(\eta_t, t)$, which is assumed to satisfy

$$F^T(\eta_t, t) F(\eta_t, t) \leq I, \quad \forall t \geq 0. \tag{3}$$

Here, the stochastic variable $\beta_1(t)$, which obeys Bernoulli distribution, is adopted to depict the corresponding ROUs and assumed to obey the following probability distribution law [9,15]:

$$\Pr(\beta_1(t)=1)=\beta_1, \quad \Pr(\beta_1(t)=0)=1-\beta_1, \tag{4}$$

where $\beta_1 \in [0, 1]$ is known constant.

Lemma 1 [34]**.** *For any real matrices E and H with appropriate dimensions and time-varying matrix $F(t)$ satisfying Eq. (3), the following inequality holds*

$$EF(t)H + H^T F^T(t)E^T < \epsilon^{-1}EE^T + \epsilon H^T H,$$

for any scalar $\epsilon > 0$.

Assumption 1. In this paper, the Semi-MJS in Eq. (1) is assumed to satisfy that

1. the state variables are not fully measurable;
2. the matrix pair $[A(\eta_t, t), B_2(\eta_t)]$ is stabilizable and detectable for $\eta_t \in \mathcal{N}$;
3. the matrix pair $[C_2(\eta_t), D_{21}(\eta_t)]$ has full-row rank for $\eta_t \in \mathcal{N}$.

2.2. Problem formulation

We will first design a DOFC for the Semi-MJS in Eq. (1), described in the following model-dependent form:

$$\begin{cases} \dot{x}_K(t) = A^K(\eta_t)x_K(t) + B^K(\eta_t)y(t), \\ u(t) = C^K(\eta_t)x_K(t), \end{cases} \tag{5}$$

where $x_K(t) \in \mathbb{R}^{n_K}$ is the state vector of the designed DOFC with $n_K = n$; $A^K(\eta_t) \in \mathbb{R}^{n\times n}$, $B^K(\eta_t) \in \mathbb{R}^{n\times q}$ and $C^K(\eta_t) \in \mathbb{R}^{m\times n}$ are the controller gains to be obtained with $\eta_t \in \mathcal{N}$.

In this paper, the phenomenon of cyber attacks has been taken into consideration in network environment and the Bernoulli process is employed to describe random network attacks. The output of the actuator is proposed as follows:

$$\bar{u}(t) = \beta_2(t)u(t) + (1-\beta_2(t))\beta_3(t)f(t), \tag{6}$$

where $f(t)$ denotes the signal from the deception attack and satisfies $\|f(t)\| \le \sigma$, where $\sigma > 0$ is a constant; $\beta_2(t)$ and $\beta_3(t)$ are Bernoulli distributed variables exhibiting the occurrence of DoS attacks and deception attacks, respectively, and $\mathbb{E}\{\beta_2(t)\} = \beta_2$, $\mathbb{E}\{\beta_3(t)\} = \beta_3$.

Remark 2. As mentioned in Eq. (6), the considered Semi-MJSs in Eq. (1) is under hybrid cyber attacks including DoS attacks and deception attacks:

1. No attack, ($\beta_2(t) = 1$ for all $\beta_3(t)$).
2. Occurrence of DoS attack, ($\beta_2(t) = 0$ and $\beta_3(t) = 0$).
3. Occurrence of deception attack, ($\beta_2(t) = 0$ and $\beta_3(t) = 1$).

For each possible value $\eta_t = i \in \mathcal{N}$, the closed-loop system resulted from combining the Semi-MJS in (1) with the DOFC in Eq. (5) can be obtained as follows:

$$\begin{cases} \dot{\xi}_1(t) = A_i^W(t)\xi_1(t) + B_i^W(t)\bar{\omega}(t), \\ z(t) = C_i^W(t)\xi_1(t) + D_i^W(t)\bar{\omega}(t), \end{cases} \tag{7}$$

where $\xi_1(t) \triangleq \begin{bmatrix} x^T(t) & x_K^T(t) \end{bmatrix}^T \in \mathbb{R}^{n+n}$, $\bar{\omega}(t) \triangleq \begin{bmatrix} \omega^T(t) & f^T(t) \end{bmatrix}^T \in \mathbb{R}^{m+l}$ and

$$\begin{cases} A_i^W(t) & \triangleq A_{0i}^W + \beta_1 \mathbf{\Delta}\tilde{A}_i(t) + (\beta_1(t) - \beta_1)\mathbf{\Delta}\tilde{A}_i(t) + (\beta_2(t) - \beta_2)\mathcal{G}(t), \ \tilde{D}_{12i} \triangleq [\ 0 \quad D_{12i}\ ], \\ B_i^W(t) & \triangleq B_{0i}^W + \Big[(\beta_2(t) - \beta_2)\beta_3 + (1-\beta_2)(\beta_3(t) - \beta_3) + (\beta_2(t) - \beta_2)(\beta_3(t) - \beta_3)\Big]\tilde{B}_{2i}, \\ C_i^W(t) & \triangleq C_{0i}^W + (\beta_2(t) - \beta_2)\tilde{C}_i, \quad C_{0i}^W \triangleq [\ C_{1i} \quad \beta_2 D_{12i} C_i^K\ ], \quad \tilde{C}_i \triangleq [\ 0 \quad D_{12i} C_i^K\ ], \\ D_i^W(t) & \triangleq D_{0i}^W + \Big[(\beta_2(t) - \beta_2)\beta_3 + (1-\beta_2)(\beta_3(t) - \beta_3) + (\beta_2(t) - \beta_2)(\beta_3(t) - \beta_3)\Big]\tilde{D}_{12i}, \\ A_{0i}^W & \triangleq \begin{bmatrix} A_i & \beta_2 B_{2i} C_i^K \\ B_i^K C_{2i} & A_i^K \end{bmatrix}, \ \mathcal{G}(t) \triangleq \begin{bmatrix} 0 & B_{2i}(t) C_i^K \\ 0 & 0 \end{bmatrix}, \ \mathbf{\Delta}\tilde{A}_i(t) \triangleq \begin{bmatrix} \mathbf{\Delta}A_i(t) & 0 \\ 0 & 0 \end{bmatrix}, \\ B_{0i}^W & \triangleq \begin{bmatrix} B_{1i} & (1-\beta_2)\beta_3 B_{2i} \\ B_i^K D_{21i} & 0 \end{bmatrix}, \ \tilde{B}_{2i} \triangleq \begin{bmatrix} 0 & B_{2i} \\ 0 & 0 \end{bmatrix}, D_{0i}^W \triangleq [\ D_{11i} \quad (1-\beta_2)\beta_3 D_{12i}\ ]. \end{cases}$$

Let us review some basic definitions, which are essential in evaluating the performance of the resultant system in (7) with the designed DOFC in Eq. (5).

Definition 1 [2]**.** The resultant closed-loop system in Eq. (7) with $\bar{\omega}(t) = 0$ is exponentially mean-square stable, if for every initial state $(\xi_1(0), \eta_0)$, the solution $\xi_1(t)$ satisfies

$$\mathbf{E}\{\|\xi_1(t)\|_2 | \xi_1(0), \eta_0\} \le \kappa \|\xi_1(0)\|_2 e^{-\psi t}, \quad \forall t \ge 0,$$

for constants $\kappa \ge 1$ and $\psi > 0$.

Motivated by [19], the supply rate associated with the resultant system in Eq. (7) is assumed to be

$$s(\bar{\omega}(t), z(t)) = z^T(t)\mathcal{Q}z(t) + 2z^T(t)\mathcal{S}\bar{\omega}(t) + \bar{\omega}^T(t)\mathcal{R}\bar{\omega}(t) \tag{8}$$

where matrices $\mathcal{Q} \in \mathbb{R}^{p\times p}$, $\mathcal{S} \in \mathbb{R}^{p\times(m+l)}$ and $\mathcal{R} \in \mathbb{R}^{(m+l)\times(m+l)}$ are known and satisfy $\mathcal{Q} = \mathcal{Q}^T < 0$ and $\mathcal{R} = \mathcal{R}^T$.

Definition 2 [19]**.** Consider the closed-loop system in Eq. (7) with the supply rate $s(\bar{\omega}(t), z(t))$ introduced in Eq. (8), a scalar $\vartheta > 0$ can be found if the following condition holds

$$\mathbf{E}\left\{\int_0^{t^*} s(\bar{\omega}(t), z(t))dt\right\} > \vartheta \mathbf{E}\left\{\int_0^{t^*} \bar{\omega}^T(t)\bar{\omega}(t)dt\right\}. \tag{9}$$

for any $t^* \ge 0$ and under zero initial condition, then the system in Eq. (7) is strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \vartheta$-dissipative. And the system in Eq. (7) is $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative, if Eq. (9) can only be satisfied for $\vartheta = 0$.

Problem Statement: Given a continuous-time Semi-MJS with ROUs and time-varying TPM in Eq. (1), we focus on the dissipativity-based DOFC design under hybrid cyber attacks ensuring that the resultant system in Eq. (7) is exponentially mean-square stable and strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative.

3. Main results

The dissipativity-based method to the analysis and synthesis of the continuos-time uncertain Semi-MJS under hybrid cyber attacks will be presented in this section.

3.1. Stability analysis

For the closed-loop system in Eq. (7), we will first develop the sufficient conditions for the exponential mean-square stability with strictly dissipative performance.

Theorem 1. *For given scalars $\epsilon_i > 0$, $\psi > 0$ and matrices $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{R}$ satisfying Eq. (8), a scalar $\vartheta > 0$ can be found if there exist symmetric matrices $P_i(k_r) > 0$ $(i \in \mathcal{N})$ satisfying*

$$\Theta_i(k_r) + H^T \left[\sum_{j=1}^{N} \alpha_{ij}(h) P_j(k_r) + \frac{dP_i(k_r)}{dt} \right] H < 0, \tag{10}$$

where

$$\begin{cases} \Theta_i(k_r) \triangleq \begin{bmatrix} \mathrm{sym}\{P_i(k_r)A_{0i}^W\} + \psi P_i(k_r) & P_i(k_r)B_{0i}^W - C_{0i}^{WT}\mathcal{S} & C_{0i}^{WT}\mathcal{Q} & P_i(k_r)\bar{M}_i & \epsilon_i \bar{N}_i^T \\ * & \vartheta I - \mathcal{R} - 2D_{0i}^{WT}\mathcal{S} & D_{0i}^{WT}\mathcal{Q} & 0 & 0 \\ * & * & \mathcal{Q} & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -\epsilon_i I \end{bmatrix}, \\ \bar{N}_i \triangleq [\, \beta_1 N_i \;\; 0 \,], \quad \bar{M}_i \triangleq [\, M_i^T \;\; 0 \,]^T, \; H \triangleq [\, I \;\; 0 \;\; 0 \;\; 0 \;\; 0 \,], \end{cases}$$

and A_{0i}^W, B_{0i}^W, C_{0i}^W, D_{0i}^W are defined in Eq. (7), then the closed-loop system in Eq. (7) is exponentially mean-square stable with strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance. Furthermore, the state decay can be estimated by

$$\mathbf{E}\{\|\xi_1(t)\|_2 | \xi_1(0), \eta_0\} \le \kappa \|\xi_1(0)\|_2 e^{-\frac{1}{2}\psi t}, \quad \forall t \ge 0,$$

where

$$\kappa \triangleq \sqrt{\frac{c}{b}} \ge 1, \quad b \triangleq \min_{\forall i \in \mathcal{N}} \lambda_{min}(P_i(k_r)), \quad c \triangleq \max_{\forall i \in \mathcal{N}} \lambda_{max}(P_i(k_r)).$$

Proof. The proof can refer to Appendix A and it is omitted here due to the limited space. □

Remark 3. It should be mentioned that the proposed performance criterion condition in this paper includes some well known results as special cases. When $\mathcal{Q} = -I$, $\mathcal{S} = 0$, $\mathcal{R} = \gamma^2 I$, the closed-loop system in Eq. (7) will be exponentially mean-square stable with $\mathcal{H}_\infty$ performance; When $\mathcal{Q} = 0$, $\mathcal{S} = I$, $\mathcal{R} = 0$, the resultant system in Eq. (7) will exhibit exponential passivity, and if $\mathcal{Q} = -\gamma^{-1}\theta I$, $\mathcal{S} = (1-\theta)I$, $\mathcal{R} = \gamma\theta I$, the resultant system in Eq. (7) will be exponentially stable with a mixed passivity and $\mathcal{H}_\infty$ performance.

It is obvious that there will be an infinite number of inequalities in Theorem 1 for the time-varying TPM $\alpha_{ij}(h)$, making it difficult and costly to design a DOFC. Now we will improve Theorem 1 by assuming that the TPs satisfy (2).

Theorem 2. *For given scalars $\epsilon_i > 0$, $\psi > 0$ and matrices $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{R}$ satisfying Eq. (8), a scalar $\vartheta > 0$ can be found if there exist symmetric matrices $P_i^r > 0$ and R_i^r $(i \in \mathcal{N})$ satisfying*

$$2\Phi_i^r + \sum_{\mathcal{N}_{uc}^i} \alpha_{ij}^r H^T (P_j^s - R_i^s) H + \sum_{\mathcal{N}_{uc}^i} \alpha_{ij}^s H^T (P_j^r - R_i^r) H < 0, \quad 1 \le r \le s \le S, \tag{11}$$

$$P_j^r - R_i^r < 0, \;\; j \in \mathcal{N}_{uk}^i, \;\; i \ne j, \tag{12}$$

$$P_j^r - R_i^r > 0, \ \ j \in \mathcal{N}_{uk}^i, \ \ i = j, \tag{13}$$

where

$$\begin{cases} \Phi_i^r & \triangleq \begin{bmatrix} \Phi_{11i}^r & P_i^r B_{0i}^W - C_{0i}^{WT}\mathcal{S} & C_{0i}^{WT}\mathcal{Q} & P_i^r \bar{M}_i & \epsilon_i \bar{N}_i^T \\ * & \vartheta I - \mathcal{R} - 2D_{0i}^{WT}\mathcal{S} & D_{0i}^{WT}\mathcal{Q} & 0 & 0 \\ * & * & \mathcal{Q} & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -\epsilon_i I \end{bmatrix}, \\ \Phi_{11i}^r & \triangleq \ \mathrm{sym}\{P_i^r A_{0i}^W\} + \psi P_i^r + \sum_{l=1}^{S-1} \pm(\delta_l)(P_i^l - P_i^S), \end{cases}$$

and A_{0i}^W, B_{0i}^W, C_{0i}^W, D_{0i}^W, $\bar{N}_i$, $\bar{M}_i$, H *are defined in Theorem 1, then the resultant system in Eq. (7) is exponentially mean-square stable with a strict* $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$*-dissipative performance.*

Proof. The proof can refer to Appendix B and it is omitted here due to the limited space. □

Remark 4. As stated in Theorem 2, the TPM of the considered Semi-MJSs in Eq. (1) is partially unknown and the sufficient conditions for the existence of the DOFC are more common for covering the completely known TPM. Besides, $\pm(\delta_l)(P_i^l - P_i^S)$ means every combinations of $\delta_l(P_i^l - P_i^S)$ and $-\delta_l(P_i^l - P_i^S)$.

3.2. Dissipativity-based DOFC design

We will solve the dissipativity-based DOFC design problem based on the result presented in Theorem 2 and propose the corresponding algorithm in this subsection.

Theorem 3. *Consider an uncertain Semi-MJS in Eq. (1). For given scalars* $\epsilon_i > 0$, $\psi > 0$ *and matrices* $\mathcal{Q}$, $\mathcal{S}$ *and* $\mathcal{R}$ *satisfying Eq. (8), if there exist symmetric matrices* $X > 0$, $Y > 0$, $\bar{P}_i^r > 0$, $\bar{R}_i^r$, *and matrices* $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$ $(i \in \mathcal{N})$ *satisfying*

$$2\Xi_i + \sum_{j\in\mathcal{N}_{uc}^i} \alpha_{ij}^r H^T(\bar{P}_j^s - \bar{R}_i^s)H + \sum_{j\in\mathcal{N}_{uc}^i} \alpha_{ij}^s H^T(\bar{P}_j^r - \bar{R}_i^r)H < 0, \quad 1 \le r \le s \le S, \tag{14}$$

$$\bar{P}_j^r - \bar{R}_i^r < 0, \ \ j \in \mathcal{N}_{uk}^i, \ \ i \neq j, \tag{15}$$

$$\bar{P}_j^r - \bar{R}_i^r > 0, \ \ j \in \mathcal{N}_{uk}^i, \ \ i = j, \tag{16}$$

where

$$\Xi_i \triangleq \begin{bmatrix} \mathrm{sym}\{\Xi_{1i}\} + \psi\Upsilon & \Xi_{2i} & \Xi_{3i} & \Xi_{4i} & \Xi_{5i} \\ * & \vartheta I - \mathcal{R} - 2D_{0i}^{WT}\mathcal{S} & D_{0i}^{WT}\mathcal{Q} & 0 & 0 \\ * & * & \mathcal{Q} & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -\epsilon_i I \end{bmatrix}, \ \Upsilon \triangleq \begin{bmatrix} X & I \\ I & Y \end{bmatrix},$$

$$\Xi_{1i} \triangleq \begin{bmatrix} A_i X + \beta_2 B_{2i}\mathcal{C}_i & A_i \\ \mathcal{A}_i & YA_i + \mathcal{B}_i C_{2i} \end{bmatrix}, \ \Xi_{3i} \triangleq \begin{bmatrix} XC_{1i}^T + \beta_2 \mathcal{C}_i^T D_{12i}^T \\ C_{1i}^T \end{bmatrix}\mathcal{Q}, \ \Xi_{4i} \triangleq \begin{bmatrix} M_i \\ YM_i \end{bmatrix},$$

$$\Xi_{2i} \triangleq \begin{bmatrix} B_{1i} & (1-\beta_2)\beta_3 B_{2i} \\ YB_{1i} + \mathcal{B}_i D_{21i} & Y(1-\beta_2)\beta_3 B_{2i} \end{bmatrix} - \begin{bmatrix} XC_{1i}^T + \beta_2 \mathcal{C}_i^T D_{12i}^T \\ C_{1i}^T \end{bmatrix}\mathcal{S}, \ \ \Xi_{5i} \triangleq \begin{bmatrix} \beta_1 X N_i^T \\ \beta_1 N_i^T \end{bmatrix}\epsilon_i,$$

and D_{0i}^{W}, H are defined in Theorem 1, then there exists a DOFC in Eq. (5) ensuring the exponential stability with a strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative performance of the resultant system in Eq. (7). And the DOFC parameters can be solved by

$$\begin{cases} A_i^K = V^{-1}(\mathcal{A}_i - YA_iX)U^{-T} - \beta_2 V^{-1}YB_{2i}C_i^K - B_i^K C_{2i}XU^{-T}, \\ B_i^K = V^{-1}\mathcal{B}_i, \quad C_i^K = \mathcal{C}_i U^{-T}, \quad VU^T = I - YX. \end{cases} \tag{17}$$

Proof. The proof can refer to Appendix C and it is omitted here due to the limited space. □

Remark 5. It is worth mentioning that by replacing $\bar{P}_j^r = \Pi_1^T P_j^r \Pi_1$, we can obtain the required DOFC parameters for the Semi-MJS as presented in Theorem 3, which brings more conservation by ignoring i and j. To handle this problem, as pointed out in [35,36], the slack Lyapunov function matrix P_i^r with a certain special structure will be introduced to obtain the DOFC parameters [37,38].

Corollary 1. *Consider an uncertain Semi-MJS in Eq. (1). For given scalars $\epsilon_i > 0$, $\psi > 0$ and matrices $\mathcal{Q}$, $\mathcal{S}$ and $\mathcal{R}$ satisfying Eq. (8), if there exist symmetric matrices $\mathcal{X} > 0$, $\mathcal{Y} > 0$, $\bar{P}_i^r > 0$, $\bar{R}_i^r$, and matrices $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$ $(i \in \mathcal{N})$ satisfying*

$$2\Xi_i + \sum_{j\in\mathcal{N}_{uc}^i} \alpha_{ij}^r H^T(\bar{P}_j^s - \bar{R}_i^s)H + \sum_{j\in\mathcal{N}_{uc}^i} \alpha_{ij}^s H^T(\bar{P}_j^r - \bar{R}_i^r)H < 0, \quad 1 \le r \le s \le S, \tag{18}$$

$$\bar{P}_j^r - \bar{R}_i^r < 0, \ \ j \in \mathcal{N}_{uk}^i, \ \ i \neq j, \tag{19}$$

$$\bar{P}_j^r - \bar{R}_i^r > 0, \ \ j \in \mathcal{N}_{uk}^i, \ \ i = j, \tag{20}$$

$$\mathcal{X}\mathcal{Y} = I, \tag{21}$$

where

$$\begin{cases} \Xi_i \triangleq \begin{bmatrix} \mathrm{sym}\{\Xi_{1i}\} + \psi\Upsilon & \Xi_{2i} & \Xi_{3i} & \Xi_{4i} & \Xi_{5i} \\ * & \vartheta I - \mathcal{R} - 2D_{0i}^{WT}\mathcal{S} & D_{0i}^{WT}\mathcal{Q} & 0 & 0 \\ * & * & \mathcal{Q} & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -\epsilon_i I \end{bmatrix}, \quad \Upsilon \triangleq \begin{bmatrix} \frac{1}{2}\mathcal{X} & I \\ I & \mathcal{Y} \end{bmatrix}, \\ \Xi_{1i} \triangleq \begin{bmatrix} \frac{1}{2}A_i\mathcal{X} + \beta_2 B_{2i}\mathcal{C}_i & A_i \\ \mathcal{A}_i & \mathcal{Y}A_i + \mathcal{B}_i C_{2i} \end{bmatrix}, \quad \Xi_{3i} \triangleq \begin{bmatrix} \frac{1}{2}\mathcal{X}C_{1i}^T + \beta_2\mathcal{C}_i^T D_{12i}^T \\ C_{1i}^T \end{bmatrix}\mathcal{Q}, \quad \Xi_{4i} \triangleq \begin{bmatrix} M_i \\ \mathcal{Y}M_i \end{bmatrix}, \\ \Xi_{2i} \triangleq \begin{bmatrix} B_{1i} & (1-\beta_2)\beta_3 B_{2i} \\ \mathcal{Y}B_{1i} + \mathcal{B}_i D_{21i} & \mathcal{Y}(1-\beta_2)\beta_3 B_{2i} \end{bmatrix} - \begin{bmatrix} \frac{1}{2}\mathcal{X}C_{1i}^T + \beta_2\mathcal{C}_i^T D_{12i}^T \\ C_{1i}^T \end{bmatrix}\mathcal{S}, \quad \Xi_{5i} \triangleq \begin{bmatrix} \frac{1}{2}\beta_1\mathcal{X}N_i^T \\ \beta_1 N_i^T \end{bmatrix}\epsilon_i, \end{cases}$$

and D_{0i}^{W}, H are defined in Theorem 1, then there exists a DOFC in Eq. (5) ensuring the exponential stability with a strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipative performance of the resultant system in Eq. (7). And the DOFC parameters can be solved by

$$A_i^K = 2\mathcal{Y}^{-1}\mathcal{A}_i\mathcal{X}^{-1} - A_i - \beta_2 B_{2i}C_i^K - B_i^K C_{2i}, \quad B_i^K = \mathcal{Y}^{-1}\mathcal{B}_i, \quad C_i^K = 2\mathcal{C}_i\mathcal{X}^{-1}. \tag{22}$$

Proof. If we choose $V = Y = \mathcal{Y}$ and $X = U = 0.5\mathcal{X}$, then we can obtain Corollary 1 and the proof is similar to Theorem 5. □

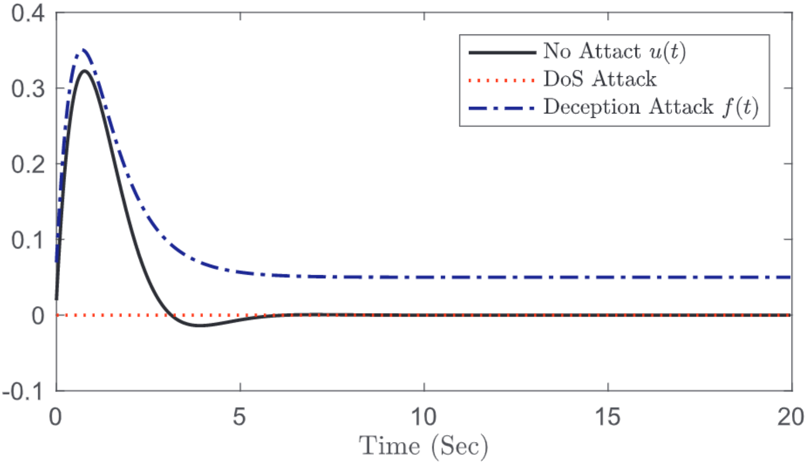


Fig. 1. Control input without attack $u(t)$, DoS attack and deception attack $f(t)$.

What should be mentioned is that by introducing a slack matrix P_i^r with a certain structure, we can reduce the number of unknown variables and achieve less conservation. However, not all the presented conditions in Theorem 3 and Corollary 1 are written in LMI form. To handle this problem, the following minimization problem is formulated.

Problem: Dissipativity-based DOFC Design

$$\min \ \text{trace}(\bar{X}Y)$$

subject to (18)-(21).

To solve the dissipativity-based DOFC design problem, Algorithm 1 is proposed utilizing the cone complementarity linearization.

4. Numerical example

To estimate the effectiveness and potential of the proposed dissipativity-based DOFC design algorithm, a numerical example on uncertain Semi-MLJS in Eq. (1) with simulation results is presented in this section. The considered continuous-time Semi-MLJS with four-subsystems is given by:

$$\left[\begin{array}{c|c|c} A_1 & B_{11} & B_{21} \\ \hline C_{11} & D_{111} & D_{121} \\ \hline C_{21} & D_{211} & \end{array}\right] = \left[\begin{array}{cc|c|c} 0.05 & 0.12 & 0.3 & 0.3 \\ -0.1 & -0.21 & 0.7 & 0.6 \\ \hline 1.3 & 0.6 & 0.3 & 0.1 \\ \hline 0.7 & 0.8 & 0.5 & \end{array}\right], \left[\begin{array}{c|c|c} A_2 & B_{12} & B_{22} \\ \hline C_{12} & D_{112} & D_{122} \\ \hline C_{22} & D_{212} & \end{array}\right]$$

$$= \left[\begin{array}{cc|c|c} -0.11 & 0.16 & 0.6 & 0.5 \\ -0.08 & 0.39 & 0.4 & 0.6 \\ \hline 1.1 & 1.6 & 0.3 & 0.1 \\ \hline 0.1 & 1.0 & 0.5 & \end{array}\right], \left[\begin{array}{c|c|c} A_3 & B_{13} & B_{23} \\ \hline C_{13} & D_{113} & D_{123} \\ \hline C_{23} & D_{213} & \end{array}\right] = \left[\begin{array}{cc|c|c} 0.55 & 0.5 & 0.2 & 0.5 \\ -0.31 & 0.0 & 1.5 & 0.4 \\ \hline 1.6 & 1.0 & 0.3 & 0.1 \\ \hline 0.8 & 0.5 & 0.5 & \end{array}\right],$$

Algorithm 1 DOFC Design by CLA.

Step 1: Provide a set of feasible initial solutions ($\mathcal{X}_0$, $\mathcal{Y}_0$, $\bar{P}^r_{0i}$, $\bar{R}^r_{0i}$, $\mathcal{A}_{0i}$, $\mathcal{B}_{0i}$, $\mathcal{C}_{0i}$) ($i \in \mathcal{N}$) satisfying (18)-(21). Set $\nu = 0$.
Step 2: Compute the following minimization problem to obtain the variables ($\mathcal{X}$, $\mathcal{Y}$, $\bar{P}^r_i$, $\bar{R}^r_i$, $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$):

$$\begin{aligned} &\min \ \operatorname{trace}(\mathcal{X}_\nu \mathcal{Y} + \mathcal{X}\mathcal{Y}_\nu) \\ &\quad \text{subject to (18)-(21).} \end{aligned}$$

Then we define f^* as the corresponding optimized value and set

$$\begin{aligned} &(\mathcal{X}_{\nu+1}, \mathcal{Y}_{\nu+1}, \bar{P}^r_{(\nu+1)i}, \bar{R}^r_{(\nu+1)i}) = (\mathcal{X}, \mathcal{Y}, \bar{P}^r_i, \bar{R}^r_i), \\ &\quad (\mathcal{A}_{(\nu+1)i}, \mathcal{B}_{(\nu+1)i}, \mathcal{C}_{(\nu+1)i}) = (\mathcal{A}_i, \mathcal{B}_i, \mathcal{C}_i). \end{aligned}$$

Step 3: If the obtained variables ($\mathcal{X}$, $\mathcal{Y}$, $\bar{P}^r_i$, $\bar{R}^r_i$, $\mathcal{A}_i$, $\mathcal{B}_i$, $\mathcal{C}_i$) are feasible for the inequality (23)-(25)

$$2\hat{\Xi}_i + \sum_{j\in\mathcal{N}^i_{uc}} \alpha^r_{ij} H^T \left(\hat{P}^s_j - \hat{R}^s_i\right) H + \sum_{j\in\mathcal{N}^i_{uc}} \alpha^s_{ij} H^T \left(\hat{P}^r_j - \hat{R}^r_i\right) H < 0, 1 \le r \le s \le S, \tag{23}$$

$$\hat{P}^r_j - \hat{R}^r_i < 0, \ j \in \mathcal{N}^i_{uk}, \ i \neq j, \tag{24}$$

$$\hat{P}^r_j - \hat{R}^r_i > 0, \ j \in \mathcal{N}^i_{uk}, \ i = j, \tag{25}$$

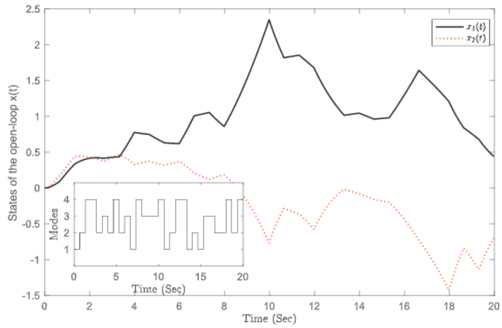
where Ξ_{3i}, Ξ_{5i}, D^W_{0i} are defined in Theorem 3, and

$$\begin{cases} \hat{\Xi}_i \triangleq \begin{bmatrix} \operatorname{sym}\left\{\hat{\Xi}_{1i}\right\} + \psi\hat{\Upsilon}_i & \hat{\Xi}_{2i} & \Xi_{3i} & \hat{\Xi}_{4i} & \Xi_{5i} \\ * & \vartheta I - \mathcal{R} - 2D^{WT}_{0i}\mathcal{S} & D^{WT}_i \mathcal{Q} & 0 & 0 \\ * & * & \mathcal{Q} & 0 & 0 \\ * & * & * & -\epsilon_i I & 0 \\ * & * & * & * & -\epsilon_i I \end{bmatrix}, \\ \hat{\Xi}_{1i} \triangleq \begin{bmatrix} \frac{1}{2}A_i\mathcal{X} + \beta_2 B_{2i}\mathcal{C}_i & A_i \\ \mathcal{A}_i & \mathcal{X}^{-1}A_i + \mathcal{B}_i C_{2i} \end{bmatrix}, \ \hat{\Xi}_{4i} \triangleq \begin{bmatrix} M_i \\ \mathcal{X}^{-1}M_i \end{bmatrix}, \ \hat{\Upsilon}_i \triangleq \begin{bmatrix} \frac{1}{2}\mathcal{X} & I \\ I & \mathcal{X}^{-1} \end{bmatrix}, \\ \hat{\Xi}_{2i} \triangleq \begin{bmatrix} B_{1i} & (1-\beta_2)\beta_3 B_{2i} \\ \mathcal{X}^{-1}B_{1i} + \mathcal{B}_i D_{21i} & \mathcal{Y}(1-\beta_2)\beta_3 B_{2i} \end{bmatrix} - \begin{bmatrix} \frac{1}{2}\mathcal{X}C^T_{1i} + \beta_2 \mathcal{C}^T_i D^T_{12i} \\ C^T_{1i} \end{bmatrix}\mathcal{S}. \end{cases}$$

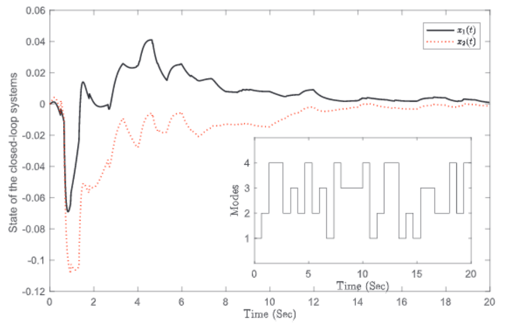
And the following condition holds

$$|f^* - 2n| < \Delta,$$

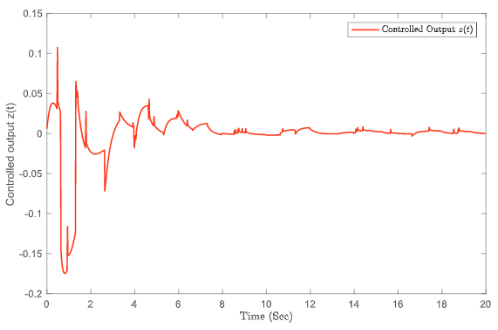
for a small scalar $\Delta > 0$, then exit.
Step 4: Otherwise, set $\nu = \nu + 1$ and$\mathcal{X}_\nu = \mathcal{X}$,$\mathcal{Y}_\nu = \mathcal{Y}$,$\hat{P}^r_{(\nu)i} = \hat{P}^r_i$,$\hat{R}^r_{(\nu)i} = \hat{R}^r_i$,$\mathcal{A}_{(\nu)i} = \mathcal{A}_i$,$\mathcal{B}_{(\nu)i} = \mathcal{B}_i$,$\mathcal{C}_{(\nu)i} = \mathcal{C}_i$, and go to Step 2.



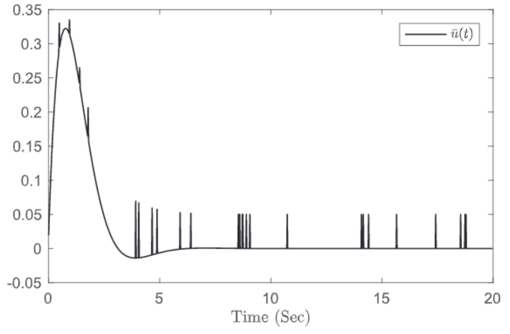
(a) State trajectories: open-loop



(b) State trajectories: closed-loop



(c) Controlled output trajectories of the closed-loop system



(d) Control input $\bar{u}(t)$

Fig. 2. Simulation Results.

$$\left[\begin{array}{c|c|c} A_4 & B_{14} & B_{24} \\ \hline C_{14} & D_{114} & D_{124} \\ \hline C_{24} & D_{214} & \end{array}\right] = \left[\begin{array}{cc|c|c} -0.34 & 0.19 & 0.5 & 0.4 \\ 0.20 & -0.62 & 0.5 & 0.5 \\ \hline 1.2 & 0.8 & 0.3 & 0.1 \\ \hline 1.0 & 0.5 & 0.5 & \end{array}\right],$$

$\epsilon_1=\epsilon_2=\epsilon_3=\epsilon_4=1.5, M_1=M_2=M_3=M_4=[0.1\ \ 0.1]^T, N_1=N_2=N_3=N_4=[0.1\ \ 0.1],$ and $F(t)=0.1\sin(t)$, $\psi=2$, $\beta_1=0.4$. It is assumed that the *sojourn time* of the considered Semi-MJL system above belongs to Weibull distribution, and the corresponding time-varying TPs matrix $\Pr(t)$ is polytopic, satisfying $\Pr(t)=\sin^2(t)\Pr^{(1)}+\cos^2(t)\Pr^{(2)}$ and

$$\Pr{}^{(1)} = \begin{bmatrix} -1.3 & 0.2 & ? & ? \\ ? & ? & 0.3 & 0.3 \\ 0.6 & ? & -1.5 & ? \\ 0.4 & ? & ? & ? \end{bmatrix}, \quad \Pr{}^{(2)} = \begin{bmatrix} -0.8 & 0.2 & ? & ? \\ ? & ? & 0.2 & 0.3 \\ 0.6 & ? & -1.2 & ? \\ 0.4 & ? & ? & ? \end{bmatrix},$$

where "?" denotes the inaccessible elements. That is, $k_1(t)=\sin^2(t)$, $k_2(t)=\cos^2(t)$ with $S=2$ and $\delta_1=\delta_2=1$.

As stated in Remark 2, there are three scenarios (No attacks, DoS attacks and Deception attacks) in the considered hybrid cyber-attack cases. To verify this part, numerical simulations have been conducted with $u(t)=e^{-t}\sin(t)$, $f(t)=e^{-t}\tanh(t)+0.05$, $\beta_2=0.98$ and $\beta_3=0.95$, and the related results have been presented in Fig. 1.

Table 1
Optimal $\vartheta_{\min}$ for various criteria (when $\beta_{\varpi} = [\ 0.4\ 0.98\ 0.95\ ]$).

$\mathcal{Q}$	$\mathcal{S}$	$\mathcal{R}$	ϑ^*	performance
-1	[2 2]	$5I$	0.0116	Strict dissipative
0	[1 1]	0	0.1617	Passivity
-1	0	$\gamma^2 I$	0.1630	$\mathcal{H}_\infty$ ($\gamma_{\min} = 0.9255$)
$-\gamma^{-1}\theta$	$[1-\theta\ 1-\theta]$	$\gamma\theta I$	0.0508	mixed $\mathcal{H}_\infty$ and passivity (Setting γ=4, $\theta = 0.8$)

In Remark 3, we have mentioned that the proposed performance criterion conditions in this paper involve some well-known results as special cases. To verify this part, simulations have been conducted and the related results have been presented in Table 1, from which we can see that the proposed DOFC design method is more general in practical applications.

The main purpose is to find an exponential DOFC of the form in Eq. (5) for the continuous-time uncertain Semi-MJS under hybrid cyber attacks ensuring the exponential stability with strictly dissipative of the resulted system in Eq. (7). By using the toolbox YALMIP of MATLAB, the required DOFC parameters can be obtained by applying Algorithm 1. Specifically, we take the case of $\mathcal{Q} = -1$, $\mathcal{S} = [2\ 2]$, $\mathcal{R} = 10I$, $\beta_{\varpi} = [0.4\ \ 0.98\ \ 0.95]$ with $\vartheta^* = 0.0116$ for example and DOFC parameters are solved:

$$\left[\begin{array}{c|c} A_1^K & B_1^K \\ \hline C_1^K & \end{array}\right] = \left[\begin{array}{cc|c} -10.502 & 0.001 & -0.011 \\ -2.189 & -1.775 & -8.013 \\ \hline 3.597 & -0.516 & \end{array}\right], \left[\begin{array}{c|c} A_2^K & B_2^K \\ \hline C_2^K & \end{array}\right] = \left[\begin{array}{cc|c} -15.189 & -0.015 & -0.008 \\ -2.585 & -3.451 & -13.522 \\ \hline 4.918 & 4.812 & \end{array}\right],$$

$$\left[\begin{array}{c|c} A_3^K & B_3^K \\ \hline C_3^K & \end{array}\right] = \left[\begin{array}{cc|c} -15.036 & -0.008 & -0.015 \\ -504.649 & -2.015 & -2.093 \\ \hline 4.894 & 3.464 & \end{array}\right], \left[\begin{array}{c|c} A_4^K & B_4^K \\ \hline C_4^K & \end{array}\right] = \left[\begin{array}{cc|c} -11.902 & -0.002 & -0.011 \\ 76.156 & -1.290 & 0.275 \\ \hline 3.744 & 0.895 & \end{array}\right].$$

To estimate the effectiveness of the proposed designed dissipativity-based DOFC design algorithm, numerous simulations on the closed-loop system with the obtained DOFC have been performed and it is shown that the resultant closed-loop system has a satisfactory performance. We choose the zero initial conditions as $x_0 = [0\ \ 0]^T$ and exogenous disturbance input as $\omega(t) = e^{-t}\sin(t)$. The corresponding state responses of the open-loop system and closed-loop system are described in Fig. 2(a) and (b), respectively. Fig. 2(c) depicts the controlled output trajectory of the closed-loop system under hybrid cyber attacks given in Fig. 2(d). It is obvious from the simulation curves of Fig. 2(b)-(c) that the resultant closed-loop system under hybrid cyber attacks, although with ROUs and partially unknown time-varying TPs, is stabilized by the designed DOFC with a good performance.

5. Conclusion

In this paper, the exponential DOFC design problem for a special class of uncertain Semi-MJSs with partially access time-varying transition probability matrix was investigated, based on dissipativity-based technique. By using mode-dependent and parameter-dependent Lyapunov-Krasoskii formulation, DOFC for the uncertain Semi-MJS was designed with the sojourn-time fractionizing technique, and the corresponding DOFC gains were solved with cone complementarity linearization algorithm. Finally, numerous simulations were performed to illustrate the feasibility and potential value of the proposed dissipativity-based DOFC design approach.

Declaration of Competing Interest

We declare that we have no financial or personal relationship with other people or organizations that can inappropriately influence our work, there is no professional or other personal interest of any nature kind in any product, service or company that could be construed as influencing the position presented in ,or the review of, the manuscript entitled.

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Appendix A

Proof of Theorem 1. To verify the exponential stability of system (7) with strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity, we need to show that the system in (7) is exponentially mean-square stable with $\bar{\omega}(t) = 0$ and satisfies the inequality in Eq. (9) under nonzero $\bar{\omega}(t) \in \mathbb{R}^l$ and zero initial condition.

1) Exponential stability ($\bar{\omega}(t) = 0$): We choose the following mode-dependent stochastic Lyapunov function:

$$V(\xi_1(t), \eta_t, k_r) \triangleq \xi_1^T(t)P(\eta_t, k_r)\xi_1(t), \tag{26}$$

where $P(\eta_t, k_r) = P^T(\eta_t, k_r) > 0$. As defined in [39–42], the infinitesimal operator $\mathcal{L}$ is introduced to denote the derivative of the above Lyapunov function $V(\xi_1(t), \eta_t, k_r)$ along the Semi-Markov chain $\{\eta_t, t > 0\}$. We then have

$$\mathcal{L}V(\xi_1(t), \eta_t, k_r) = \lim_{\triangle \to o} \frac{\mathbf{E}\{V(\xi_1(t+\triangle), \eta_{t+\triangle}, k_r(t+\triangle))|\xi_1(t), \eta_t, k_r(t)\} - V(\xi_1(t), \eta_t, k_r(t))}{\triangle},$$

where $\triangle > 0$ is defined to be a small number and we obtain that

$$\mathcal{L}V(\xi_1(t), \eta_t, k_r) = \xi_1^T(t)\Gamma_i(k_r)\xi_1(t)$$

where $\Gamma_i(k_r) \triangleq \mathrm{sym}\{P_i(k_r)A_i^W(t)\} + \sum_{j=1}^N \alpha_{ij}(h)P_j(k_r) + \frac{dP_i(k_r)}{dt}$. For any $t^* > 0$, we apply Dynkin's formula

$$\begin{aligned} &\mathbf{E}\left\{e^{\psi t^*}V(\xi_1(t^*), \eta_{t^*}, k_r^*)\right\} - \mathbf{E}\left\{V(\xi_1(0), \eta_0, k_r^0)\right\} \\ &= \mathbf{E}\left\{\int_0^{t^*} e^{\psi t}(\mathcal{L}V(\xi_1(t), \eta_t, k_r) + \psi V(\xi_1(t), \eta_t, k_r)dt\right\}, \end{aligned}$$

where ψ is a positive parameter to be determined. Furthermore,

$$\mathbf{E}\left\{\int_0^{t^*} e^{\psi t}(\mathcal{L}V(\xi_1(t), \eta_t, k_r) + \psi V(\xi_1(t), \eta_t, k_r)dt\right\} \leq a\mathbf{E}\left\{\int_0^{t^*} e^{\psi t} \parallel \xi_1(t^*) \parallel^2 dt\right\},$$

where $a \triangleq \psi\lambda_{\max}(P_i(k_r)) - \lambda_{\max}(-\Gamma_i(k_r))$, we can choose the appropriate $\psi > 0$ such that $a < 0$. And note from Eq. (26) that

$$\mathbf{E}\{V(\xi_1(t), \eta_t, k_r)\} \geq b\mathbf{E}\{\| \xi_1(t) \|^2\}, \ \ b \triangleq \min_{\forall i \in \mathcal{N}} \lambda_{min}(P_i(k_r)),$$
$$\mathbf{E}\{V(\xi_1(0), \eta_0, k_r^0)\} \leq c\mathbf{E}\{\| \xi_1(0) \|^2\}, \ \ c \triangleq \max_{\forall i \in \mathcal{N}} \lambda_{max}(P_i(k_r)).$$

That is,

$$b\mathbf{E}\{\| \xi_1(t) \|^2\} \leq \mathbf{E}\{V(\xi_1(t), \eta_t, k_r)\} \leq ce^{-\psi t} \| \xi_1(0) \|^2,$$

which implies

$$\mathbf{E}\{\| \xi_1(t) \|_2\} \leq \sqrt{\frac{c}{b}} e^{-\frac{1}{2}\psi t} \| \xi_1(0) \|_2,$$

holds for any $t > 0$. Therefore, according to Definition 1, the resultant system in Eq. (7) is exponentially mean-square stable with $\bar{\omega}(t) = 0$.

2) Strict $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipativity (with zero initial condition): Here, we choose the same stochastic Lyapunov function in Eq. (26) and obtain its derivative with $\eta_t = i$ as follows:

$$\begin{aligned}
\mathcal{L}V(\xi_1(t), \eta_t, k_r) = & \lim_{\triangle \to o} \frac{1}{\triangle} \Bigg[\sum_{j=1, j\neq i}^{N} \Pr(\eta_{t+\triangle} = j | \eta_t = i) \xi_1^T(t+\triangle) P_j(k_r(t+\triangle)) \xi_1(t+\triangle) \\
& - \xi_1^T(t) P_i(k_r(t)) \xi_1(t) + \Pr(\eta_{t+\triangle}=i|\eta_t=i) \xi_1^T(t+\triangle) P_i(k_r(t+\triangle)) \xi_1(t+\triangle) \Bigg] \\
= & \lim_{\triangle \to o} \frac{1}{\triangle} \Bigg[\sum_{j=1, j\neq i}^{N} \frac{q_{ij}(G_i(h+\triangle) - G_i(h))}{1 - G_i(h)} \xi_1^T(t+\triangle) P_j(k_r(t+\triangle)) \xi_1(t+\triangle) \\
& - \xi_1^T(t) P_i(k_r(t)) \xi_1(t) + \frac{1 - G_i(h+\triangle)}{1 - G_i(h)} \xi_1^T(t+\triangle) P_i(k_r(t+\triangle)) \xi_1(t+\triangle) \Bigg],
\end{aligned}$$

where q_{ij} represents the probability intensity of the Semi-MJS jumping from subsystem i to subsystem j; $G_i(h)$ refers to the cumulative distribution function of the *sojourn-time* h when the Semi-MJS stays in subsystem i. The first-order approximation of $\xi_1(t+\triangle)$ is given by

$$\xi_1(t+\triangle) = \xi_1(t) + \triangle \dot{\xi}_1(t) + o(\triangle) = \begin{bmatrix} I + \triangle A_i^W(t) & \triangle B_i^W(t) \end{bmatrix} \zeta(t) + o(\triangle),$$

where $\triangle$ is a small scalar and $\zeta(t) \triangleq \begin{bmatrix} \xi_1^T(t) & \bar{\omega}^T(t) \end{bmatrix}^T$. Thus,

$$\begin{aligned}
& \mathcal{L}V(\xi_1(t), \eta_t, k_r) \\
& = \lim_{\triangle \to o} \frac{1}{\triangle} \Bigg\{ \sum_{j=1, j\neq i}^{N} \frac{q_{ij}(G_i(h+\triangle) - G_i(h))}{1 - G_i(h)} \zeta^T(t) \begin{bmatrix} I + \triangle A_i^{WT}(t) \\ \triangle B_i^{WT}(t) \end{bmatrix} P_j(k_r) \begin{bmatrix} I + \triangle A_i^{WT}(t) \\ \triangle B_i^{WT}(t) \end{bmatrix}^T \zeta(t) \\
& + \frac{1 - G_i(h+\triangle)}{1 - G_i(h)} \zeta^T(t) \begin{bmatrix} I + \triangle A_i^{WT}(t) \\ \triangle B_i^{WT}(t) \end{bmatrix} P_i(k_r) \begin{bmatrix} I + \triangle A_i^{WT}(t) \\ \triangle B_i^{WT}(t) \end{bmatrix}^T \zeta(t) - \xi_1^T(t) P_i(k_r) \xi_1(t) \Bigg\}.
\end{aligned}$$

Using the condition that $\lim_{\triangle\to o}\frac{G_i(h+\triangle)-G_i(h)}{1-G_i(h)}=0$, we obtain that

$$\begin{aligned}&\mathcal{L}V(\xi_1(t),\eta_t,k_r)\\&=\lim_{\triangle\to o}\Bigg\{\xi_1^T(t)\Bigg[\sum_{j=1,j\neq i}^{N}\frac{q_{ij}(G_i(h+\triangle)-G_i(h))}{\triangle\,(1-G_i(h))}P_j(k_r)+\frac{G_i(h)-G_i(h+\triangle)}{\triangle\,(1-G_i(h))}P_i(k_r)\Bigg]\xi_1(t)\\&\quad+\frac{1-G_i(h+\triangle)}{1-G_i(h)}\zeta^T(t)\begin{bmatrix}\mathrm{sym}\{P_i(k_r)A_i^W(t)\}+\frac{dP_i(k_r)}{dt} & P_i(k_r)B_i^W(t)\\ B_i^{WT}(t)P_i(k_r) & 0\end{bmatrix}\zeta(t)\Bigg\}.\end{aligned}$$

Motivated by [2], we use

$$\lim_{\triangle\to o}\frac{1-G_i(h+\triangle)}{1-G_i(h)}=1,\ \lim_{\triangle\to o}\frac{G_i(h+\triangle)-G_i(h)}{\triangle\,(1-G_i(h))}=\alpha_i(h),$$

where $\alpha_i(h)$ is defined to be the TPM of the Semi-MJS jumping from subsystem i. Furthermore, we have

$$\alpha_{ij}(h)\triangleq\alpha_i(h)q_{ij}\quad\text{for } j\neq i,\ \text{and } \alpha_{ii}(h)\triangleq-\sum_{j=1,j\neq i}^{N}\alpha_{ij}(h).$$

Then we obtain that

$$\mathcal{L}V(\xi_1(t),\eta_t)=\zeta^T(t)\hat{\Gamma}_i(t)\zeta(t), \tag{27}$$

where

$$\hat{\Gamma}_i(t)\triangleq\begin{bmatrix}\mathrm{sym}\{P_i(k_r)A_i^W(t)\}+\sum_{j=1}^{N}\alpha_{ij}(h)P_j(k_r)+\frac{dP_i(k_r)}{dt} & P_i(k_r)B_i^W(t)\\ * & 0\end{bmatrix}.$$

On the other hand, taking the expectations of $A_i^W(t)$, $B_i^W(t)$, $C_i^W(t)$, $D_i^W(t)$ and using Eq. (4), yields

$$\mathbf{E}\Big\{\big[A_i^{WT}(t)\ \ B_i^{WT}(t)\ \ C_i^{WT}(t)\ \ D_i^{WT}(t)\big]^T\Big\}=\mathbf{E}\Big\{\big[A_{0i}^{WT}+\beta_1\mathbf{\Delta}\tilde{A}_i^T(t)\ \ B_{0i}^{WT}\ \ C_{0i}^{WT}\ \ D_{0i}^{WT}\big]^T\Big\}.$$

To establish the strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity of the system in Eq. (7), the following performance index with nonzero $\bar{\omega}(t)\in\mathbb{R}^l$ is constructed:

$$\begin{aligned}&J(\xi_1(t),\eta_t)=\mathbf{E}\Big\{\int_0^\infty e^{\psi t}\big[\vartheta\bar{\omega}^T(t)\bar{\omega}(t)-s(\bar{\omega}(t),z(t))\big]dt\Big\}\\&=\mathbf{E}\Big\{\int_0^\infty e^{\psi t}\big[\vartheta\bar{\omega}^T(t)\bar{\omega}(t)-s(\bar{\omega}(t),z(t))+\mathcal{L}V(\xi_1(t),\eta_t)+\psi V(\xi_1(t),\eta_t)\big]dt\Big\}\\&\quad-\mathbf{E}\{e^{\psi t}V(\xi_1(t),\eta_t)\}\\&\leq\mathbf{E}\Big\{\int_0^\infty e^{\psi t}\big[\vartheta\bar{\omega}^T(t)\bar{\omega}(t)-s(\bar{\omega}(t),z(t))+\mathcal{L}V(\xi_1(t),\eta_t)+\psi V(\xi_1(t),\eta_t)\big]dt\Big\}.\end{aligned} \tag{28}$$

Then, combining Eqs. (27),(28) yields

$$J(\xi_1(t),\eta_t)\leq\mathbf{E}\Big\{\int_0^\infty\zeta^T(t)e^{\psi t}\Psi_i(t)\zeta(t)dt\Big\}, \tag{29}$$

where

$$\Psi_i(t) \triangleq \begin{bmatrix} \mathrm{sym}\{P_i(k_r)A_{0i}^W\} + \psi P_i(k_r) + \sum_{j=1}^N \alpha_{ij}(h)P_j(k_r) + \frac{dP_i(k_r)}{dt} & P_i(k_r)B_{0i}^W \\ * & -\mathcal{R} + \vartheta I \end{bmatrix}$$
$$-\begin{bmatrix} 0 & C_{0i}^{WT} \\ * & D_{0i}^{WT} + D_{0i}^W \end{bmatrix}\mathcal{S} - \begin{bmatrix} C_{0i}^{WT} \\ D_{0i}^{WT} \end{bmatrix}\mathcal{Q}\begin{bmatrix} C_{0i}^{WT} \\ D_{0i}^{WT} \end{bmatrix}^T$$
$$+\begin{bmatrix} P_i(k_r)\bar{M}_i \\ 0 \end{bmatrix}F_i(t)\begin{bmatrix} \bar{N}_i & 0 \end{bmatrix} + \begin{bmatrix} \bar{N}_i^T \\ 0 \end{bmatrix}F_i^T(t)\begin{bmatrix} \bar{M}_i^T P_i(k_r) & 0 \end{bmatrix}.$$

Applying the Schur complement and Lemma 1, we can obtain from (10) that $\Psi_i(t) < 0$, which implies $J(\xi_1(t), \eta_t) < 0$ for all nonzero $\bar{\omega}(t) \in \mathbb{R}^l$. That is,

$$\mathbf{E}\left\{\int_0^{t^*} e^{\psi t}\left(s(\bar{\omega}(t), z(t)) - \vartheta\bar{\omega}^T(t)\bar{\omega}(t)\right)dt\right\} > 0,$$

which satisfies (9). Therefore, the strictly $(\mathcal{Q}, \mathcal{S}, \mathcal{R})$-dissipative performance of the resultant closed-loop system in Eq. (7) is guaranteed. This completes the proof.

Appendix B

Proof of Theorem 2. Note that

$$P_i(k_r(t)) = \sum_{r=1}^S k_r(t)P_i^r, \Longrightarrow \frac{dP_i(k_r)}{dt} = \sum_{r=1}^S \dot{k}_r(t)P_i^r, \qquad \sum_{r=1}^S k_r(t) = 1, \Longrightarrow \sum_{r=1}^S \dot{k}_r(t) = 0,$$

we obtain that

$$\sum_{r=1}^S \dot{k}_r(t)P_i^r = \sum_{l=1}^{S-1} \dot{k}_l(t)P_i^l + \dot{k}_S(t)P_i^S = \sum_{l=1}^{S-1} \dot{k}_l(t)(P_i^l - P_i^S) = \sum_{l=1}^{S-1} \pm(\delta_l)(P_i^l - P_i^S),$$

Inspired by [9], we rewrite the inequality in Eq. (10) by replacing the term $\frac{dP_i(k_r)}{dt}$ by $\sum_{l=1}^{S-1} \pm(\delta_l) \times \left(P_i^l - P_i^S\right)$, and we can obtain

$$\Xi_i(k_r) \triangleq \sum_{r=1}^S k_r^2(t)\hat{\Xi}_i^{rr} + \sum_{r=1}^{S-1}\sum_{s=r+1}^S k_r(t)k_s(t)(\hat{\Xi}_i^{rs} + \hat{\Xi}_i^{sr}) < 0, \tag{30}$$

where

$$\hat{\Xi}_i^{rs} \triangleq \Theta_i^r + H^T\left[\sum_{j=1}^N \alpha_{ij}^r P_j^s + \sum_{l=1}^{S-1} \pm(\delta_l)(P_i^l - P_i^S)\right]H,$$

and Eq. (30) will be hold when

$$\hat{\Xi}_i^{rs} + \hat{\Xi}_i^{sr} < 0, \quad 1 \le r \le s \le S. \tag{31}$$

Considering the TPs are partially unknown and $\sum_{j=1}^N \alpha_{ij}(h) = 0$, the inequality in Eq. (31) can be rewritten as

$$2\Theta_i^r + 2\sum_{l=1}^{S-1} \pm(\delta_l)H^T(P_i^l - P_i^S)H + \sum_{j=1}^N \alpha_{ij}^r H^T P_j^s H - \sum_{j=1}^N \alpha_{ij}^r H^T R_i^s H$$

$$
\begin{aligned}
&+\sum_{j=1}^{N}\alpha_{ij}^{s}H^{T}P_{j}^{r}H-\sum_{j=1}^{N}\alpha_{ij}^{s}H^{T}R_{i}^{r}H\\
&=2\left[\Theta_{i}^{r}+\sum_{l=1}^{S-1}\pm(\delta_{l})H^{T}(P_{i}^{l}-P_{i}^{S})H\right]+\sum_{\mathcal{N}_{uc}^{i}}\alpha_{ij}^{r}H^{T}(P_{j}^{s}-R_{i}^{s})H\\
&+\sum_{\mathcal{N}_{uk}^{i}}\alpha_{ij}^{r}H^{T}(P_{j}^{s}-R_{i}^{s})H+\sum_{\mathcal{N}_{uc}^{i}}\alpha_{ij}^{s}H^{T}(P_{j}^{r}-R_{i}^{r})H+\sum_{\mathcal{N}_{uk}^{i}}\alpha_{ij}^{s}H^{T}(P_{j}^{r}-R_{i}^{r})H<0.
\end{aligned}
$$

By fractionalizing the time-varying sojourn-time h into r sections, the uncertain Semi-MJS in Eq. (1) could be treated as r individual Semi-MJS with the corresponding TPM in each section. For $\alpha_{ij}^{r}\geq 0$ $(j\in\mathcal{N}_{uk}^{ir}, i\neq j)$ and $\alpha_{i}^{r}=-\sum_{j=1,j\neq i}^{N}\alpha_{ij}^{r}<0$, we obtain Eqs. (11) (12) and (11)(13) hold, respectively. Then, we can obtain $\Psi_i(t)<0$, ensuring the exponentially mean-square stability with strict $(\mathcal{Q},\mathcal{S},\mathcal{R})$-dissipativity performance of the resultant system in Eq. (7). This completes the proof.

Appendix C

Proof of Theorem 3. According to the result presented in Theorem 2, it is obvious that there exists a DOFC introduced in Eq. (5) for the uncertain Semi-MJS in Eq. (1) ensuring the exponential stability with a strict dissipative performance of the resultant system in Eq. (7), if there exist symmetric matrices $P_i^r>0$ satisfying inequalities Eqs. (11)-(13).

Partitioning the matrices P_i^r and $(P_i^r)^{-1}$ as a special structure:

$$
P_{i}^{r}\triangleq\begin{bmatrix}Y & V\\ * & W_{i}^{r}\end{bmatrix},\quad (P_{i}^{r})^{-1}\triangleq\begin{bmatrix}X & U\\ * & Z_{i}^{r}\end{bmatrix},
$$

where $Y\in\mathbb{R}^{n\times n}$, $V\in\mathbb{R}^{n\times n}$, $W_i^r\in\mathbb{R}^{n\times n}$, $X\in\mathbb{R}^{n\times n}$, $U\in\mathbb{R}^{n\times n}$, $Z_i^r\in\mathbb{R}^{n\times n}$. Without loss of generality, it is assumed that Y and X are symmetric positive definite matrices. To linearize the nonlinear condition in Eq. (11), we define the following nonsingular matrices

$$
\Pi_{1}\triangleq\begin{bmatrix}X & I\\ U^{T} & 0\end{bmatrix},\quad \Pi_{2}\triangleq\begin{bmatrix}I & Y\\ 0 & V^{T}\end{bmatrix},
$$

it is easy to see that $P_i^r(P_i^r)^{-1}=I$, which implies $P_i^r\Pi_1=\Pi_2$. Furthermore we define

$$
\begin{cases}
\mathcal{A}_{i} \triangleq YA_{i}X+\beta_{2}YB_{2i}C_{i}^{K}U^{T}+VB_{i}^{K}C_{2i}X+VA_{i}^{K}U^{T},\\
\mathcal{B}_{i} \triangleq VB_{i}^{K},\quad \mathcal{C}_{i}\triangleq C_{i}^{K}U^{T},\quad VU^{T}\triangleq I-YX.
\end{cases}
$$

Then, we have

$$
\begin{aligned}
&\Pi_{1}^{T}P_{i}^{r}\Pi_{1}=\begin{bmatrix}X & I\\ I & Y\end{bmatrix},\ \Pi_{1}^{T}P_{i}^{r}A_{0i}^{W}\Pi_{1}=\begin{bmatrix}A_{i}X+\beta_{2}B_{2i}\mathcal{C}_{i} & A_{i}\\ \mathcal{A}_{i} & YA_{i}+\mathcal{B}_{i}C_{2i}\end{bmatrix},\ \Pi_{1}^{T}(P_{i}^{l}-P_{i}^{S})\Pi_{1}=0,\\
&\Pi_{1}^{T}P_{i}^{r}B_{0i}^{W}=\begin{bmatrix}B_{1i} & (1-\beta_{2})\beta_{3}B_{2i}\\ YB_{1i}+\mathcal{B}_{i}D_{21i} & Y(1-\beta_{2})\beta_{3}B_{2i}\end{bmatrix},\quad \Pi_{1}^{T}(P_{j}^{s}-R_{i}^{s})\Pi_{1}=\bar{P}_{j}^{s}-\bar{R}_{i}^{s},\\
&\Pi_{1}^{T}P_{i}^{r}\bar{M}_{i}=\begin{bmatrix}M_{i}\\ YM_{i}\end{bmatrix},\quad \Pi_{1}^{T}\bar{N}_{i}^{T}=\begin{bmatrix}\beta_{1}XN_{i}^{T}\\ \beta_{1}N_{i}^{T}\end{bmatrix},\quad \Pi_{1}^{T}C_{0i}^{WT}=\begin{bmatrix}XC_{1i}^{T}+\beta_{2}\mathcal{C}_{i}^{T}D_{12i}^{T}\\ C_{1i}^{T}\end{bmatrix}.
\end{aligned}
$$

Next, we develop a congruence transformation on Eqs. (11)-(13) with diag$\{\Pi_1, I, I, I, I\}$, then Θ_i^r will be rewritten as Eqs. (14)-(16). This completes the proof.

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