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An efficient hybrid metaheuristics optimization technique applied to the AC electric transmission network expansion planning

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***Abstract*—The transmission network expansion planning (TNEP) problem consists of determining the necessary infrastructure additions, within a planning horizon, to minimize an investment objective function while meeting some operational and physical constraints. Even using simplified models to represent the electric network, the TNEP becomes a very complex, combinatorial and non-convex optimization problem. In recent years, the full alternating current (AC) network model has been proposed to formulate the TNEP problem. Due to its complexity, more robust and efficient optimization techniques to solve the AC formulation are required. This paper proposes a new effcient hybrid metaheuristic technique to solve the TNEP problem. Additionally, it presents a comprehensive comparative study including different powerful conventional, emerging and hybrid optimization metaheuristics techniques applied to solve the static, long-term TNEP problem, using the AC model, considering both operating and reactive power compensation costs. Simulation results are shown for three test systems: Garver 6-bus system, IEEE 24-bus system and the IEEE 118-bus system.**

***Index Terms*—AC Model, Electric Energy Systems, Hybrid-metaheuristics, Optimization, Parallel Processing, Transmission Network Expansion Planning.**

I. Introduction

An electric energy system has to generate, transmit and distribute electrical energy to supply the demand at every instant of time, meeting operational constraints. To achieve that objective in a long-term horizon, the transmission network expansion planning (TNEP) plays a fundamental role, since it allows to determine the new transmission circuits to will be added to the electrical system, in an optimal way [1].

The TNEP problem can be studied from different points of view. An electricity market-oriented TNEP approach is developed in [1], [2]. Uncertainty in the TNEP assumptions is handled in [3]–[5]. In [6], the problem is treated using multi-objective formulations. Static and multistage TNEP formulations are presented in [7], [8] and [9], [10], respectively. From a mathematical perspective, all TNEP approaches require two main components: *i)* a mathematical model to represent the electric network and *ii)* a solution technique applied to solve the mathematical model [11]. In the specialized literature, there are different mathematical models to deal with the TNEP problem, from simplified models such as the transportation model [12], hybrids models [13] and the DC model [3], [14], to full non-linear models such as the AC model [7], [9]–[11], [15].

There are several optimization techniques to solve different optimization problems generally classified into two groups: mathematical programming and metaheuristic techniques [1], [16]. The mathematically based approaches find a solution using a calculation procedure based on research operations theory providing good quality solutions only when dealing with convex problems; although, it presents certain disadvantages due mainly to convergence problems when using complex models at very large computational time [16]. On the other hand, the metaheuristic techniques follow an iterative process of generating, evaluating and selecting candidate solutions, following logical rules. Some disadvantages of metaheuristics techniques are the need of tuning parameters and an initial population generation process. Although the solutions obtained do not guarantee to be the global optimum, these techniques allow studying problems of great complexity more simply, obtaining solutions of good quality in reasonable computational time [17]. Besides, researchers seek to improve the search capability of metaheuristics through hybridization, in such a way to combine the advantages of different metaheuristics, while simultaneously minimizing occasional disadvantages [18].

Currenly, several metaheuristic optimization techniques have already been successfully applied to the TNEP problem using simplified models, such as Genetic Algorithm [19], [20], Differential Evolution [21], Particle Swarm Optimization [22], Harmony Search [23], Tabu Search [24], Ant Colony [25], Modified Gray Wolf Optimization Algorithm [26], Hybridization of Biased Random-Key Genetic Algorithm with Local Branching [27], and Hybrid Bat-Inspired Algorithm [28].

In this research work, a new hybrid metaheuristic has been proposed to solve the TNEP problem in a very efficient and robust fashion. This metaheuristic is a combination between the Differential Evolution (DE) and the Continuous Population Based Incremental Learning (PBILc) metaheuristics to perform a more efficient search and thus cover certain disadvantages of each metaheuristic. It is worth pointing out that the application of metaheuristics techniques using the AC model to solve the TNEP problem is still in an early stage, and very few tests have been performed in different scenarios for challenging test systems [1], [7], [29]–[31].

In summary, this research work tries to fill some gaps proposing the following contributions: *i)* a new hybrid metaheuristic to solve the AC TNEP problem, *ii)* a performance comparison among different traditional metaheuristics such as Genetic Algorithm (GA), Differential Evolution (DE), Particle Swarm Optimization (PSO), Evolutionary Programming (EP), new emerging metaheuristics (never used in TNEP problems so far) such as Teaching-Learning-Based Optimization (TLBO), Big Bang-Big Crunch (BB-BC) algorithm, Continuous Population Based Incremental Learning (PBILc), Evolutionary programming integrated with Cultural Algorithm (CAEP), and three hybrid-metaheuristics (DE-PBILc, CE-PSO and EPSO) applied to solve the TNEP problem using the AC model, *iii)* the inclusion of power generation costs in the objective function and the co-optimization of shunt compensation, and *iv)* full robustness and efficiency tests using challenging test systems with existing and new demanding scenarios, including dispatchable and non-dispatchable power generation.

The remaining of this paper is organized as follows. Section II presents the mathematical formulation used to solve the TNEP problem. Section III explains the basics of the different metaheuristics proposed in this research work. Section IV shows the implementation of the proposed metaheuristics to solve the TNEP problem. Section V presents simulation results using three test systems. Section VI presents a summary of the results obtained. Finally, Section VII presents conclusions and future research works.

II. Mathematical formulation

The mathematical model used in this paper to solve the TNEP based on the AC model has divided into two problems: *i)* the expansion master problem and *ii)* the operational problem. The formulation of the expansion master problem (1) allows minimizing the total costs of transmission line additions, power generation, and load shedding.

$$min\ v = \sum_{(k,l)\in\Omega} c_{kl}\cdot n_{kl} + w \tag{1}$$

subject to

$$0 \le n_{kl} \le \overline{n_{kl}}; \quad n_{kl}\ integer \tag{2}$$

Where v is the objective function. The first term of (1) considers the cost of added transmission circuits, where c_{kl} corresponds to the cost of a circuit that can be added between the buses k and l, n_{kl} is the total number of circuits (existing and added) between the buses k and l. w is the cost of active and reactive load shedding; also, it includes the annual operating cost of existing generators. $\overline{n_{kl}}$ is the maximum number of circuits allowed between the buses k and l, Ω is the set of all rights of way where it is possible to add new transmission lines.

The operational problem corresponds to the formulation of an AC optimal power flow, with operational constraints of the system (3-12).

$$min\ w = \sum_{(k\in\wedge)} (\alpha_1\cdot r_{Pk} + \alpha_2\cdot r_{Qk} + c_{op}) \tag{3}$$

subject to

$$P(V,\theta) - P_G + P_D - r_P = 0 \tag{4}$$

$$Q(V,\theta) - Q_G + Q_D - r_Q = 0 \tag{5}$$

$$\underline{P}_G \le P_G \le \overline{P}_G \tag{6}$$

$$\underline{Q}_G \le Q_G \le \overline{Q}_G \tag{7}$$

$$\underline{r}_P \le r_P \le \overline{r}_P \tag{8}$$

$$\underline{r}_Q \le r_Q \le \overline{r}_Q \tag{9}$$

$$\underline{V} \le V \le \overline{V} \tag{10}$$

$$S^{from} \le \overline{S} \tag{11}$$

$$S^{to} \le \overline{S} \tag{12}$$

The term c_{op} represents the annual operating cost of existing generators, associated with the level of the generation that allows supplying the total demand at the lowest possible cost. c_{op} is represented mathematically by

$$c_{op} = 8,760 \sum_{(k)\in\wedge} (\beta_k\cdot P_{Gk} + \gamma_k\cdot Q_{Gk})\cdot CF_k \tag{13}$$

Where $8,760$ is the number of hours in a year, β_k corresponds to the power generation cost (or operating cost) at node k ($\$/MWh$); P_{Gk} is the total power generated at node k; CF_k corresponds to the capacity factor of the generator at node k; γ_k corresponds to the reactive power generation cost at node k ($\$/MVARh$). In this paper, the operating cost of the existing reactive power generation is zero (i.e., $\gamma_k = 0$), where Q_{Gk} is the total reactive power generated at node k and $\wedge$ is the set of all load nodes. α_1 is the cost of the active load shedding; r_P is the active load shedding; α_2 is the cost of shunt compensation; r_Q is the reactive load shedding, which in this formulation also represents the reactive power compensation needed in some buses. $\overline{P}_G$, $\overline{Q}_G$, and $\underline{P}_G$, $\underline{Q}_G$, are the vectors of maximum and minimum real and reactive power limits of existing generators, respectively. $\overline{r}_P$, $\overline{r}_Q$ and $\underline{r}_P$, $\underline{r}_Q$ are the maximum and minimum limits of active and reactive power of fictitious generators. V is the vector of voltage magnitudes with the maximum $\overline{V}$ and minimum $\underline{V}$ limits of 105% and 95% of the nominal value, respectively. θ is the phase angle vector. P_G and Q_G are the existing real and reactive power generation vectors. P_D and Q_D are the real and reactive power demand vectors. S^{from}, S^{to} and $\overline{S}$ are the apparent power flow vectors (MVA) through the branches in both terminals and their limits, respectively.

Equations (14) and (15) represent the power flow AC formulation, where elements of vectors $P(V,\theta)$ and $Q(V,\theta)$ appear in (5) and (6) and correspond to the nodal active and reactive power balances.

$$P_k(V,\theta) = V_k\cdot\sum_{k\in M} V_l\cdot[G_{kl}\cdot cos\theta_{kl} + B_{kl}\cdot sin\theta_{kl}] \tag{14}$$

$$Q_k(V,\theta) = V_k\cdot\sum_{k\in M} V_l\cdot[G_{kl}\cdot sin\theta_{kl} - B_{kl}\cdot cos\theta_{kl}] \tag{15}$$

S^{from} and S^{to} can be calculated as follow:

$$S_{kl}^{from} = \sqrt{(P_{kl}^{from})^2 + (Q_{kl}^{from})^2} \quad (16)$$

$$S_{kl}^{to} = \sqrt{(P_{kl}^{to})^2 + (Q_{kl}^{to})^2} \quad (17)$$

The calculation method of G_{kl}, B_{kl}, P_{kl}^{from}, P_{kl}^{to}, Q_{kl}^{from} and Q_{kl}^{to} can be found in [15].

In the operational problem ((3)-(12)), the shunt compensation is modelled by the reactive load shedding ($\alpha_2 \cdot r_{Qk}$). The cost of shunt compensation is a linear function of the variable cost, which provides approximated costs for both capacitive and inductive reactive power needed in load nodes. The cost of shunt compensation is always positive, so when capacitive compensation (positive power injection) is obtained, the cost coefficient α_2 is modelled as positive, while when inductive compensation (negative power injection) is obtained, the coefficient of cost α_2 is modelled as negative. A further explanation on shunt compensation can be found in [7], [29].

The load shedding cost corresponds to a strategy intended to penalize the objective function using fictitious generators of active and reactive power that must be added in all load nodes (PQ nodes) in case the existing generators or a certain transmission topology can not supply the entire demand (load not served or not attended) [7]. Therefore, it also helps the optimization algorithms to converge more easily. In a certain iteration i of the metaheuristic algorithm, some generated transmission topologies (as many as the size of the population) are evaluated through an AC optimal power flow (3 - 12). If a particular transmission topology is feasible, the AC optimal power flow will converge even in case of constraint violation. A constraint violation is initially met by the AC optimal power flow through generation redispatch (which is the most economical way). If this is not possible, the AC optimal power flow will use fictitious generators (the most expensive way). In this case, the following situations may arise [7]:

1. There is no active power load shedding ($r_P = 0$). It means that the fictitious generators do not generate active power. This situation is only possible when the current topology satisfies the system's active power needs and, there are no constraint violations. However, it does not necessarily mean that the current topology is the optimal one since in further iterations more economical plans that also meet all the constraints may be found.

2. There is an active power load shedding ($r_P > 0$). It means that the fictitious generators do generate active power and some constraint violations are present. In case this option is allowed, the production cost (α_1) of the fictitious generators can be set, for instance, as the actual cost of the energy not supplied or a cost higher than the current transmission expansion cost of that topology (say the cost of the most expensive transmission plan). By setting the fictitious generators with the actual cost of the energy not supplied, we are accepting that the final transmission plan could have some load shedding. Setting that cost to a much higher value is a way to penalise (1), and the optimization algorithm in further iterations will discriminate those "expensive" plans. This is, in turn, a way of rejecting load shedding in the final plan.

3. There is no reactive load shedding ($r_Q = 0$). It means that the fictitious generators do not generate reactive power. Therefore, there is no constraint violation related to reactive power. This condition can be achieved in two situations. The first one is the case where the current transmission topology provides the reactive power paths with the load needs, so there is no need for additional reactive power sources in the system. The second situation is related to defining unbounded reactive power sources. It is possible to achieve this condition by setting the production cost α_2 of the fictitious generators to zero ($\alpha_2 = 0$).

4. There is reactive load shedding. Reactive power is produced when there is some related constraint violation. It happens when the current transmission topology does not meet the reactive power requirements of the system, so there is a need for reactive power compensation ($r_Q > 0$). This condition requires that the cost of shunt compensation is always positive. In this case, the coefficient cost is $\alpha_2 > 0$ in case of capacitive compensation and $\alpha_2 < 0$ in case of inductive compensation.

If a certain transmission topology is not feasible, the AC optimal power flow of the operational problem will not converge. Even when the formulation of this problem is defined having in mind the creation of feasible solutions due to the load shedding approach, sometimes unfeasible solutions can occur mainly if the power generation range of the fictitious generators is too strict. In that case, a way of avoiding unfeasible solutions is to set large power generation limits. Additionally, some unfeasible solutions can appear when scenarios without allowing shunt compensation are analysed because there is no support of reactive power from the fictitious generators ($r_Q = 0$) and the current transmission topology is not able to handle it. Therefore, in general, the way to tackle unfeasible solutions is to set w in (1) to a very high value to penalise the objective function and discriminate that topology.

III. Metaheuristic techniques

A. Differential evolution (DE)

This algorithm is based on populations, where each individual within the population undergo cross over and mutation before competing with individuals in the current generation. The steps of the DE are explained in Algorithm 1. DE has already been applied to solve the TNEP problem. More detail of this metaheuristic technique applied to the TNEP problem used in this paper can be found in [29].

B. Continuous Population Based Incremental Learning (PBILc)

The PBILc is based on competitive learning. At each generation, it determines both the most relevant and irrelevant characteristics of individuals in each population, and it learns from these characteristics to generate new individuals through

Algorithm 1 Pseudocode of the DE algorithm.

Step 1: Initial population. Randomly initialize a initial population $x^0 = [x_1^0, x_2^0, \ldots x_i^0, \ldots, x_m^0]$ of m individuals, where each individual is represent by $x_i^0 = [x_{i,1}^0, \ldots, x_{i,j}^0, \ldots x_{i,n}^0]$, where n corresponds to the dimension of the problem.

while the stopping criterion is not satisfied **do**

for $i = 1 : m$ **do** (m is the size of the population)

Step 2: Differential mutation. Generate a donor vector v_i^k from the a perturbation in each individual i:

$$v_{i,j}^k = x_b^k + F \cdot (x_{1,j}^k - x_{2,j}^k + x_b^k - x_{3,j}^k) \tag{18}$$

Step 3: Crossover. Generate a trial vector u_i^k from the information sharing of the individuals of the current population x_i^k and the donor vector v_i^k:

if $r < Cr \mid J = j_{rand}$ **then**

$$u_{i,j}^k = v_{i,j}^k \tag{19}$$

else

$$u_{i,j}^k = x_{i,j}^k \tag{20}$$

end if

Step 4: Selection. Compare the trial vector u_i^k with each individual of the current population x_i^k, to determine which individual survives to the next generation:

if $f(u_i) < f(x_i)$ **then**

$$x_i^{k+1} = u_i^k \tag{21}$$

else

$$x_i^{k+1} = x_i^k \tag{22}$$

end if

end for

end while

*$F \in [0, 2]$ is the mutation rate, $C_r \in [0, 2]$ is the crossover factor, x_b^k is the individual with the best solution, $J = j_{rand}$ is a random value between $[0, n]$, and it guarantees that at least one component of the donor vector v_i^k exchanges information to u_i^k. $x_1^k, x_2^k, x_3^k \in [1, m]$ with $x_1 \neq x_2 \neq x_3$.

sampling a probability vector. The steps of the PBILc are explained in Algorithm 2 [32].

C. Hybridization of Differential Evolution and PBILc (DE-PBILc)

This metaheuristic is a combination of the Differential Evolution (DE) and the Continuous Population Based Incremental Learning (PBILc) metaheuristics. It aims to combining DE (which uses the difference of the parameter vectors to explore the search space) and PBILc (which employs probabilistic models based on competitive learning) to perform a more efficient search and thus to cover certain disadvantages of each metaheuristic. The traditional selection and crossover operators are not effective enough to obtain optimal or near-optimal solutions and exhibit poor performance in high dimensional problems. In this new hibrid metaheuristic, a double differential mutation is used to accelerate the optimization process without losing robustness or efficiency. To start the optimization process, a combination probability ($p_{comb} \in [0, 1]$) is established. Each individual is generated using DE if a randomly generated number r (generated between $[0, 1]$) does not exceed the p_{comb}. Otherwise, the individual will be generated using PBILc (see Algorithm 3). Additionally, when DE is used to generate an individual, at the time of applying the differential mutation, it is possible to decide between two types of differential mutation based on the decision whether a randomly generated number does not exceed the probability of mutation ($p_{double-mut} \in [0, 1]$). The two differential mutation applied are: $DE/best/2$ strategy and trigonometric mutation [33].

Algorithm 2 Pseudocode of the PBILc metaheuristic.

Step 1: Initialize initial population (similar to the DE) and probabilistic model. The n-dimensional probabilistic model p used is based on a normal distribution function model $p(\mu, \sigma)$, so the value of the initial mean $p(\mu^0) = \mu_1^0, ..., \mu_n^0$ is generated from a normal random distribution within the search domain $[x^{min}, x^{max}]$ and the initial standard deviation $p(\sigma^0) = \sigma_1^0, ..., \sigma_n^0$ is set in order to obtain diversity of individuals.

while the stopping criterion is not satisfied **do**

Step 2: Update probability vector. The probability vector $p(\mu^k, \sigma^k)$ will be updated based on:

for $j = 1 : n$ **do**

$$\mu_j^k = (1 - \eta) \cdot \mu_j^k + \eta \cdot (x_{(1,j)}^k + x_{(2,j)}^k - x_{(1,j)}^k) \tag{23}$$

$$\sigma_i^k = (1 - \eta) \cdot \sigma_i^k + \eta \sqrt{\sum_{i=1}^{N_{best}} \frac{(x_{(i,j)} - \mu_{best}^k)^2}{N_{best}^k}} \tag{24}$$

end for

Step 3: Generate the next population. The next generation is generated using a random normal distribution:

for $i = 1 : m$ **do** (m is the size of the population)

$$x_{i,j}^{k+1} = N(p(\mu_j^k), p(\sigma_j^k)) \tag{25}$$

end for

end while

*η ($[0, 1]$) is called learning rate. N_{best} represents the number of individuals with the best solutions, μ_{best} is mean of the N_{best} individuals, and $x_{(1,j)}^k$, $x_{(2,j)}^k$ represent the first two individuals with the best solutions.

D. Genetic Algorithm (GA)

The GA is an optimization technique inspired by imitating the genetic processes of living beings [34]. Such optimization technique is widely implemented on various optimization problems and specifically to solve the TNEP problem (using simplified models) obtaining good quality results [19], [27]. GA begins with an initial population of randomly generated individuals. Within the population, each individual is assigned

Algorithm 3 Pseudocode of the DE-PBILc metaheuristic.

Step 1: Initial population (similar to the DE).
while the stopping criterion is not satisfied **do**
  for $i = 1 : m$ **do** (m is the size of the population)
    if $r \leftarrow N[0,1] < p_{comb}$ **then**
      Step 2: Apply double differential mutation.
      $x_1^k, x_2^k, x_3^k, x_4^k \rightarrow$ selected randomly
      if $r \leftarrow N[0,1] < p_{double-mut}$ **then**

$$v_{i,j}^k = \frac{x_{1,j}^k - x_{2,j}^k - x_{3,j}^k}{3} + (p_2 - p_1) \cdot (x_{1,j}^k - x_{2,j}^k) + (p_3 - p_2) \cdot (x_{2,j}^k - x_{3,j}^k) + (p_1 - p_3) \cdot (x_{3,j}^k - x_{1,j}^k) \quad (26)$$

      else

$$v_{i,j}^k = x_b^k + F \cdot (x_{1,j}^k - x_{2,j}^k + x_b^k - x_{3,j}^k) \quad (27)$$

      end if
      Step 3: Apply Crossover. Section III-A
    else
      Step 4: Apply PBILc (Generate individual. Section III-B)
    end if
    Step 5: Apply Selection (section III-A)
  end for
  Step 6: Update probability vector. Section III-B
end while

$p_1 = f(x_1)^k/p'$; $p_2 = f(x_2)^k/p'$; $p_3 = f(x_3)^k/p'$ and $p' = f(x_1) + f(x_2) + f(x_3)$; x_b^k is the individual with the best solution.

with an aptitude value (in this work the aptitude value is the objective function value). Subsequently, the individuals with the best aptitude values are more likely to be selected (using selection by a tournament with probability [34]) to share the genetic information to the following generations through the crossing (using uniform crossing [34]) and mutation operators (using uniform distribution [34]).

E. Big Bang-Big Crunch Algorithm (BB-BC))

This algorithm is inspired by the theories of the universe (cosmological theories), from the birth of the universe in the big bang to the death of the same in the big crunch. Details of this metaheuristic can be found in [35].

F. Evolutionary programming (EP)

EP is inspired in the theory of evolution, where the next generation is created only by mutation (no recombination applies) [34]. EP has already been applied to solve the TNEP problem. Details and settings of this metaheuristics applied to the TNEP problem can be found in [36].

G. Cultural Algorithm (CA)

The cultural algorithm is based on the idea of the cultural evolution of a society, noting that the population evolves from parents to children and also evolves culturally since culture can be transmitted to the next generation as a hereditary process [34]. The previous idea was taken to introduce in the evolutionary process of the EP the cultural algorithm (named CAEP). CAEP has already been applied to solve the TNEP problem. Details and settings of this metaheuristics applied to the TNEP problem can be found in [36].

H. Particle Swarm Optimization (PSO)

PSO is an optimization technique inspired by the social behaviour of the movement (direction and speed) the flight of flocks of birds. PSO (version LPSO) has already been applied to solve the TNEP problem. Details and settings of this metaheuristics applied to the TNEP problem can be found in [7].

I. Hybrid Evolutionary Particle Swarm Optimization (EPSO

EPSO was developed to overcome difficulties related to the premature convergence and slow finish that are common in Genetic Algorithm (GA) and Particle Swarm Optimization (PSO). The hybrid algorithm combines concepts of evolutionary computation and multiagent population taking advantage of the standard blocks that are typical in GA and PSO techniques. Also, the Hill-Climbing method is used to improve the exploitation in the search space. EPSO has already been applied to solve two different approaches to TNEP with very promising results. Details and settings of this metaheuristics applied to the TNEP problem can be found in [37], [38].

J. Cross-Entropy Method and Evolutionary Particle Swarm Optimization (CE-PSO)

CE-PSO is a combination of the Cross-Entropy (CE) method (exploration) and the Evolutionary Particle Swarm Optimization (exploitation) to improve state exploration and exploitation. Additionally, this metaheuristic presents a simple iterative algorithm (based on the Two-way Analysis of Variance (ANOVA)) which is used to fine-tune EPSO‘s strategic parameters to consistently obtain close-to-optimal solutions. This metaheuristic has already been applied to solve the TNEP problem where a special recombination operator for handling transmission network investment decisions in EPSO also is presented. Details and settings of this metaheuristics applied to the TNEP problem can be found in [37].

IV. TNEP IMPLEMENTATION

This section describes the implementation of the metaheuristics described in the previous section to solve the TNEP problem using the AC model. Figure 1 shows the flow chart of the implementation.

1) *Network data:* Network data are the initial system data. The dimension of the problem n is given by the number of right-of-ways, where it is possible to add circuits. The initial topology is x^{min} and the maximum number of circuits per right-of-way is x^{max}.
2) *General parameters:* These parameters are necessary for the convergence of the algorithm, as the number of individuals of the population m, the maximum number of iterations or generations I_{max}.
3) *Parameters of metaheuristics:* The parameters of the metaheuristics that have been already applied to solve

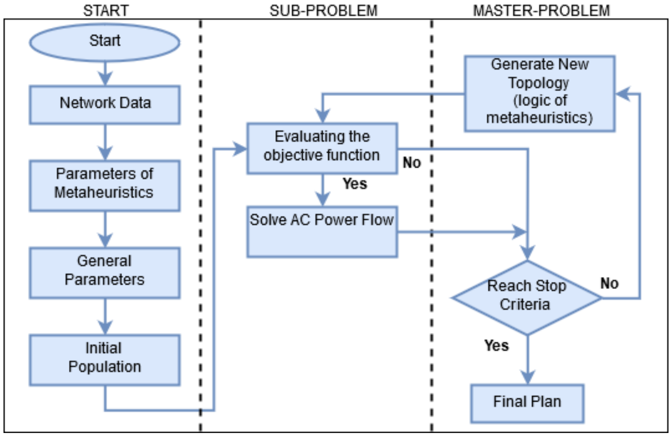

Fig. 1. *Flow chart for solving the TNEP.*

the TNEP problem were taken from their respective references, while for the rest of the metaheuristics the parameters were established based on a trial and error method to obtain the best results for most tests. Table I shows the parameters used by the different metaheuristics for all test systems.

4) *Initial population:* The initial population is randomly generated.
5) *Reducing the evaluations of the objective function:* This method was implemented only to the DE-PBILc metaheuristic and is based on employing less optimal power flow and speed up the optimization process since the greatest computational effort in solving the TNEP problem is given in solving the optimal power flow formulation (3-12) [29]. The main idea behind this improvement is that in a first stage it is enough to know the cost of each topology (only lines cost) without the need to solving the optimal power flow. If each topology i (test vector) presents a lower value (only lines cost), this topology will calculate an optimal power flow. Otherwise, it will be penalized with a high value in such a way to consider a topology with a poor solution in the selection step (section III-C step 5).
6) *New population:* The new population generated at each iteration by the different metaheuristics is limited within the search domain (initial topology x^{min} and the maximum allowed number of circuits per right-of-way x^{max}).
7) *Population evaluation:* In this step, each individual (topology) is evaluated to determine the value of the objective function. The population is evaluated in parallel using the ”Matlab Parallel Computing Toolbox” to solve simultaneous operational problems. In this case, each worker of the parallel toolbox performs a certain number of optimal power flows, which speeds up the whole process of evaluating the function.
8) *Stopping Criterion:* In this paper, for all the proposed metaheuristics, the maximum allowed number of iterations was used as a stopping criterion.

V. Results

The performance of the different metaheuristics to solve the TNEP problem is evaluated. Three test systems are used, namely Garver 6-bus system, IEEE 24-bus system and 118-bus system. To compare the performance of the different metaheuristics and the quality of their results, three main criteria were considered. The first one is the success rate, which is an indicator of robustness since it determines the percentage of runs for which the optimal value is obtained. In this research work, at least 70% of success rate is considered as a reasonable robustness indicator. The second criterion is based on the standard deviation calculated from ten simulations, which is indicative of the convergence variability (high or low random search component) of the metaheuristic. The third one corresponds to the average number of iterations, which is a statistical average of the number of iterations required in each simulation until a local minimum, the global minimum or a known reference value are obtained. The latter demonstrates the metaheuristic’s efficiency. Besides, the best and worst (the minimum and maximum cost of the objective function for all the tests, respectively) solutions found are presented. In this research work, load shedding is not allowed in the final plans. The metaheuristics were implemented in MATLAB [39], running on an Intel i5, 3.1 GHz, 8GB RAM, hardware platform. MATPOWER [40] is used to solve the AC optimal power flow formulation ((4)-(12)). The parameters used in the different metaheuristics are shown in Table I. Different test scenarios have been defined for each test system to test the feasibility of this approach:

a) Unlimited shunt compensation: in this scenario, the reactive power generation limits of fictitious generators are set in a wide range, where the reactive power cost $\alpha_2 = 0$. In this case, the reactive power generation can be dispatched freely whenever needed.

b) No shunt compensation: in this case, the reactive power generation limits of fictitious generators are set to zero.

c) Limited shunt compensation: this scenario allows reactive power generation from the fictitious generators within a predefined range. The generated reactive power cost is set to a nonzero value.

d) Considering the active power cost in the objective function: in this case, the active power cost of existing generators is included in the objective function using cost β_k.

Other scenarios can be created by setting the annual operating cost β_k in scenarios *a)*, *b)* and *c)* above. The test systems are considered for both dispatchable generation and non-dispatchable generation. Furthermore, simulations using the DC model are also performed in such a way as to compare the results obtained.

A. Garver 6-bus system

This system has 6 buses, 15 candidate branches, with a total power demand of 760MW and 152MVAr, 1,100MW of maximum power generation and the maximum allowed number of circuits per right-of-way is five. The complete data can be found in [14]. As for the scenarios considering operating

Table I
Parameters of metaheuristics.

Metaheuristic	GA	BB-BC	EPSO	PBILc	EP	CAEP	DE	LPSO	DE-PBILc	CE-PSO
Parameters	$p_m = 0.03$ $p_{cross} = 0.75$ $p_c = 0.5$ $p_{selec} = 0.7$ $m_{elite} = 1$	$d_1 = 1$	$n_{hillC} = m/4$ $n_{hc} = m/4$ (small systems) $n_{hc} = 10$ (big systems) $p_{elite} = 0.6$ $r = 3$ $\chi = 0.729$ $c_1 = 2.05$ $c_2 = 2.05$	$\eta = 0.05$ $N_{best} = m/2$ μ^0 randomly $\sigma^0 = 2$ $m_{elite} = 1$	$\beta = 0$ $\gamma = 0.2$	$\beta = 0$ $\gamma = 0.2$ $B = [x^{min}, x^{max}]$ $M = 12m\%$ $\alpha = 0.5$	$F = 0.7$ $Cr = 0.6$	$\chi = 0.729$ $k_1 = 2.05$ $k_2 = 2.05$ $NR = 1$	$F = 1$ $Cr = 0.2$ $\eta = 0.05$ $N_{best} = m/2$ μ^0randomly $\sigma^0 = 2$ $m_{elite} = 1$ $p_{double-mut} = 0.3$ $p_{comb} = 0.9$	$N_{elite} = 20$ $\sigma = 0.9$ $p_{comunication} = 0.8$ $r_{mutation} = 0.4$ $index_{Ro} = m - N_{elite}$ $max\ iter_{ce} = 20$ $init = 1$ $weights = rand(m * 0.25, 4) * 1.5$

Table II
TNEP results for Garver system with dispatchable generation.

Added lines	Scenarios							
	DC1	A1				A2	A3	A4
2-3	0	0	0	0	0	0	2	2
2-6	0	0	1	2	3	2	2	2
3-5	1	1	1	1	1	2	3	4
4-6	3	3	2	1	0	2	3	4
Total Cost (M$)	110	110	110	110	110	160	30,395.32	30,428.14
Total Active Power Generation (MW)	760	779.89	774.64	774.85	781.48	772.23	766.51	766.18
Total Active Power Gen. Cost (M$)	-	-	-	-	-	-	30,145.32	30,128.14
Cost Lines (M$)	110	110	110	110	110	160	250	300
Shunt Comp.	-	Yes	Yes	Yes	Yes	No	Yes	No
Total Shunt Comp. (MVAr)	-	251.46	197.07	183.61	260.05	-	191.03	-
Bus 2 Comp. (MVAr)	-	96.69	70.04	51.73	98.57	-	70.75	-
Bus 4 Comp. (MVAr)	-	82.21	54.31	57.23	83.24	-	51.5	-
Bus 5 Comp. (MVAr)	-	72.54	72.36	74.64	78.23	-	68.76	-

costs, the capacity factors are $CF_1 = 0.6$, $CF_3 = 0.6$, and $CF_6 = 0.7$. The operating costs ($\$/MWh$) of each generator were set to: $\beta_1 = 0.005$, $\beta_3 = 0.007$ and $\beta_6 = 0.0085$.

1) Dispatchable generation: For this case, four scenarios were considered: *i)* scenario with unlimited shunt compensation at load nodes (A1), where the shunt compensation limits were set from -1,000 to 1,000MVAr, *ii)* scenario without allowing shunt compensation (A2), *iii)* and *iv)* scenario A3 and A4 similar to scenario A1 and A2 but considering the power generation cost. The results for each scenario are listed in Table II. Additionally, the final plan using the DC model was obtained (DC1). The tests were performed for a population of 60 individuals, 150 iterations allowed and performing 10 tests.

For the base case (scenario A1), four final topologies present the smallest value found with a total investment cost of $110M for each of them. For each one of the four topologies a total shunt compensation of 251.46MVAr, 197.07MVAr, 183.61MVAr and 260.05MVAr is required, respectively. The result where the lines $l_{3-5} = 1$ and $l_{4-6} = 3$ are added agree with the topology obtained in [7], [14], but the other three additional topologies are feasible too. This scenario shows that the four topologies can be considered as the optimal final topology. The scenario A2 presents a greater total investment cost ($160M) concerning the scenarios A1 ($110M). This higher cost is because the system does not allow to add compensation shunt. For the scenario A3, incorporating the operating cost, the final plan presents a total investments cost of $30,395.32M, where $250M corresponds to lines investments and $30,145.32M corresponds to the operating cost. Also, the obtained plan requires shunt compensation of 191.03MVAr, located at nodes 2, 4 and 5 with 70.75, 51.5 and 68.76MVAr, respectively. On the other hand, scenario A4 presents a total investments cost of $30,428.14M, where $300M corresponds to lines investments and $30,128.14M corresponds to the operating cost. The previous results indicate that the overall cost has two components, namely due to the expansion itself and due to the further operation of the system. Therefore, in some situations, the obtained expansion plan may exhibit a higher cost than of other expansion plans. However, the operational cost is low, in such a way that the final plan could be beneficial, presenting a higher cost by added transmission circuits, but savings due to less operational cost. On the other hand, the final plan obtained using the DC model matches with that of the literature [14], [20] and this final plan is exactly the same final plan obtained using the AC model (with added lines in $l_{3-5} = 1$ and $l_{4-6} = 3$), however, the AC model provides three additional final plans to the previous one with their respective shunt compensation.

Table III
Success rates of the metaheuristics for Garver system (scenario A).

Algorithm	GA	BB-BC	PBILc	EP	CAEP	DE	LPSO	DE-PBILc	CE-PSO	EPSO
Scenario	Success Rate (%)									
A1	30	60	100	100	100	100	100	100	100	100
A2	60	30	100	100	100	100	100	100	100	100
A3	100	0	100	100	100	100	90	100	100	100
A4	100	0	100	100	100	100	100	100	100	100

Table III shows the success rates of the metaheuristics for each scenario A analyzed, where the majority of metaheuristics are robust (success rates $> 90\%$), except the metaheuristics GA and BB-BC, which for some cases does not present reasonable robustness (scenario A1 and A2) and even the BB-BC cannot find the minimum value found by the rest of metaheuristics (scenarios A3 and A4). Table IV shows the performance results of metaheuristics for scenarios A1 and A3. Table IV shows that among the robust metaheuristics, the DE metaheuristic is the one that requires the least number of evaluations of the objective function (Eval.F.O). Also, EPSO requires a low number of iterations to converge (< 27 iterations). GA and PBILc require the greatest number of iterations (> 68 iterations) to find the minimum value (see Fig. 2). In addition, almost all metaheuristics have low convergence variability (Standard Deviation of < 12). Only GA and CE-PSO presents high convergence variability (Standard Deviation of 39 and 22, respectively).

The advantage in terms of the computational time from the use of parallel processing is shown in Table V. For parallel processing, 2 and 4 workers are used. The results in the Table V show that as the number of workers increases, the computa-

Table IV

Metaheuristics performance for Garver test system, scenario A1 and A3, with a population of 60 individuals, 150 iterations allowed and performing 10 tests.

Scenario	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3	A1	A3
Metaheuristic	**GA**		**BB-BC**		**PBILc**		**EP**		**CAEP**		**DE**		**LPSO**		**DE-PBILc**		**CE-EPSO**		**EPSO**	
Success Rate (%)	40	100	60	10	100	100	100	100	100	100	100	100	100	90	100	100	100	100	100	100
Eval. F.O	4,177	3,445	500	420	4,158	4,476	2,904	3,288	1,952	2,178	752	1,494	1,350	1,873	887	2,322	1,272	1,197	1,541	5,208
Stand. Deviat. Iter.	39	7	1	0	10	12	10	10	4	4	5	3	5	6	5	4	22	10	2	11
Average Iter.	68	57	8	7	69	75	48	55	33	36	25	25	22	31	34	39	32	30	6	27
Lowest Cost (M$)	110	30,395.32	110	30,490.3	110	30,395.32	110	30,395.32	110	30,395.32	110	30,395.32	110	30,395.32	110	30,395.32	110	30,395.32	110	30,395.32
Higher Cost (M$)	148	30,395.32	151	30,694.8	110	30,395.32	110	30,395.32	110	30,395.32	110	30,395.32	110	30,396.11	110	30,395.32	110	30,395.32	110	30,395.32
Cost Lines (M$)	110	250	110	211	110	250	110	250	110	250	110	250	110	250	110	250	110	250	110	250
Total Gen. (MW)	774.64	774.64	774.64	769.08	774.64	774.64	774.64	774.64	774.64	766.51	774.64	766.51	774.64	766.51	774.64	766.51	774.64	766.51	774.64	766.51
Gen. Cost (M$)	-	30,145.32	-	30,279.3	-	30,145.32	-	30,145.32	-	30,145.32	-	30,145.32	-	30,145.32	-	30,145.32	-	30,145.32	-	30,145.32

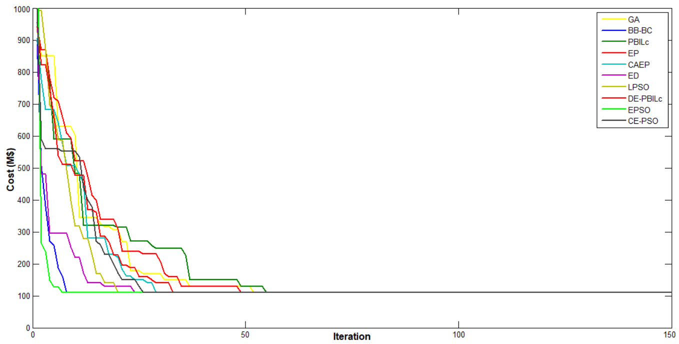

Fig. 2. *Convergence process for scenario A1 with 60 individuals.*

tional time required is further reduced; for example, the LPSO metaheuristic using serial processing requires 5.06min to solve the TNEP problem and as the number of internal workers increases, the computational time required is further reduced (2.4min and 1.48min with 2 and 4 workers, respectively).

2) Non-dispatchable generation: In this case, two scenarios were considered: *i)* shunt compensation with 10,000$/MVAr (B1) in nodes 2, 4 and 5, and *ii)* the last one scenario (B2) similar to the previous one but considering the power generation cost. Shunt compensation limits were set at -1,000 and 1,000 MVAr. Additionally, the final plan using the DC model was obtained (scenario DC2). Table VI shows the final plan found for each scenario. The tests were performed for a population of 60 individuals, 200 iterations allowed and performing 10 tests.

Scenario B1 presents a final plan with an investment cost of $170.63M, with a cost of $170M in transmission circuits additions, similar to the previous scenario, but with a shunt compensation cost of $0.63M. The total amount of shunt compensation is 63.96MVAr in nodes 2, 4 and 5 with 12, 6.14 and 45.7MVAr, respectively. For scenario B2, the final plan presents a total investments cost of $36,483.4M, where $341M corresponds to lines investments and $36,142.4M corresponds to the operating cost. The previous results indicate that the overall cost has two components, namely due to the expansion itself and due to the further operation of the system. Therefore, in some situations, the obtained expansion plan may exhibit a higher cost than of other expansion plans. However, the operational cost is low, in such a way that the sum of the two terms results in overall optimal value.

The final plan obtained using the DC model matches with that of the literature [14], [20], however, this plan (with $200M) is more expensive than the scenario using the AC model (scenario B1 with $170.63M), since the apparent power flow limits in transmission lines for the AC model are higher than the correspondingly active power limits of the DC model. Also, this difference is explained by the unlimited reactive power compensation allowed in all load nodes.

Table VII shows the performance results of metaheuristics for scenarios B1 and B2, where only EP, CAEP, DE and DE-PBILc are considered robust (success rates $>= 70\%$), highlighting among them the CAEP, DE and DE-PBILc metaheuristics with a success rate of the 100%. In another hand, again GA and BB-BC present poor robustness (low success rate). Moreover, the success rate has been reduced for some metaheuristics (with respect to scenario A), which is because the problem with non-dispatchable generation is a problem of greater difficulty. One of the metaheuristics that presents a notable reduction in robustness is CE-PSO, which had a success rate of 100% for scenario A, while for scenario B its success rate is 40%. Additionally, Table VII shows that robust DE is the one that requires the least number of evaluations of the objective function (Eval.F.O). Also, another efficient metaheuristic, CAEP, requires between 34-35 iterations to find the minimum value, while GA and PBILc require the largest number of iterations (>65 iterations) to find the minimum value. In addition, almost all of the most robust metaheuristics EP, CAEP, DE and DE-PBILc present low convergence variability (Standard Deviation < 12), except EP which presents high convergence variability.

B. IEEE 24-bus system

The system consists of 24 buses and 41 rights-of-way. The maximum allowed number of circuits per right-of-way is five, with a total power demand of 8,550MW and 1,740MVAr. The complete data for the system can be found in [14]. The scenario with unlimited shunt compensation was considered for both dispatchable and non-dispatchable generation (C1 and C2, respectively), and the last two scenarios similar to the previous ones but considering the power generation cost (scenarios C3 and C4). As for scenarios considering

Table V
Computing time for Garver system (scenario A1) using parallel processing.

Numbers of workers		Metaheuristic									
		GA	BB-BC	PBILc	EP	CAEP	DE	LPSO	DE-PBILc	CE-PSO	EPSO
1 (Serial)	**Time (min)**	4.7	4.6	5	5.08	4.8	1.08	5.06	1.3	2.9	15.4
2		2.46	4.58	2.45	5.05	2.3	0.8	2.4	1	1.58	9.2
4		1.5	4.43	1.46	4.7	1.48	0.65	1.48	0.8	1.3	5.9

Table VI
Expansion plans for Garver system with non-dispatchable generation.

Added lines	Scenarios		
	DC2	**B1**	**B2**
2-6	4	3	5
3-5	1	1	2
4-6	2	2	3
5-6	0	0	1
Total Cost (M$)	200	170.63	36,483.4
Total Power Gen. (MW)	760	784.39	773.36
Total Cost Power Gen. (M$)	-	-	36,142.4
Cost Lines (M$)	200	170	341
Total Shunt Comp. (MVAR)	-	63.96	0
Comp. cost (M$)	-	0.63	0

operating costs, a capacity factor $CF = 0.6$ was set for all generators. The operating costs ($\$/MWh$) of each generator were set to: $\beta_1 = 0.002$, $\beta_2 = 0.0016193$, $\beta_7 = 0.0012468$, $\beta_{13} = 0.0013928$, $\beta_{15} = 0.0016193$, $\beta_{16} = 0.001236$, $\beta_{18} = 0.0014253$, $\beta_{21} = 0.0014253$, $\beta_{22} = 0.0016193$ and $\beta_{23} = 0.0011892$. Additionally, the final plan using the DC model was obtained for both dispatchable and non-dispatchable generation (DC3 and DC4, respectively).

The final plans for each of those scenarios are shown in Table VIII. The tests were performed for a population of 60 individuals, 500 iterations allowed and performing 10 tests. The results obtained for the scenario C1 presents a total investment cost of \$48M (only lines). This final plan requires a total compensation of 1,993.89MVAr. On the other hand, when considering the generation cost (scenario C3), the final plan presents a total cost of \$63,162.97M, where \$580M represents the investments in line additions and \$62,582.97M represents the operating cost. The results obtained for the scenario C2 presents a total investment cost of \$98M (only lines), and when considering the generation cost (scenario C4), the final plan presents a total cost of \$65,667.07M, where \$254M represents the investments in line additions and \$65,413.07M represents the generation cost. Previous results indicate that, when considering the cost of operation, the final plan could be beneficial, presenting a higher cost by added transmission circuits, but savings due to less operational cost.

The final plan obtained using the DC model matches with that of the literature [14], [20], however, this plan (with \$152M) is more expensive than the scenario using the AC model (scenario C1 with \$48M). A similar result is presented for the non-dispatchable generation, where the final plan for the scenario DC4 (with \$256M) is more expensive than the scenario C2 with an investment of \$98M. The lower cost of the resulting plan for the AC model compared to that of the DC model is due to the allowed unlimited reactive power compensation in all load nodes and the apparent power flow limits in transmission lines, which are higher than the

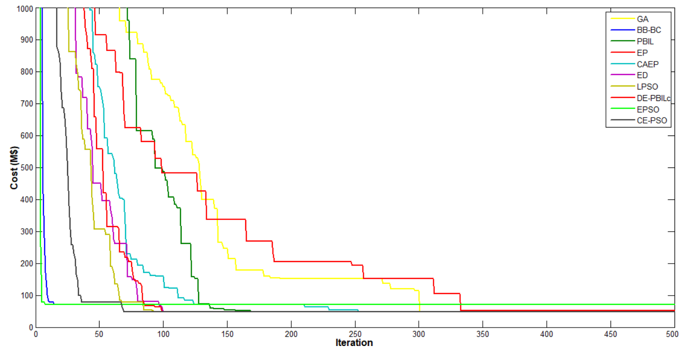

Fig. 3. *Convergence process of metaheuristics for scenario C1.*

corresponding active power limits of the DC model.

Table IX shows the success rates of the metaheuristics for each scenario C analyzed, where most metaheuristics have reduced their robustness if compared to the Garver 6-bus system (Fig. III), and only DE and DE-PBIL are considered robust (success rate $> 70\%$). The most significant reduction in robustness is presented in EP since this metaheuristic is robust on low dimensionality systems (Garver 6-bus system with the success rates $> 70\%$), but as the dimensionality grows this metaheuristic reduces its robustness (IEEE 24-bus system with the success rates $= 0\%$).

Table X shows the performance results of metaheuristics for scenarios C1 and C3. Table X shows that DE and DE-PBILc (robust metaheuristics) present similar efficiency for scenario C1 since they require between 92 and 100 iterations until finding the minimum value, but in scenario C2, DE-PBILc requires only 116 iterations to find the minimum value while DE requires 199 iterations. Therefore, DE-PBILc is proved again as a robust and efficient optimization technique. Also, DE-PBILc presents a low convergence variability (Standard Deviation < 13). The convergence process of one test for scenario C1 is shown in Fig. 3, where although EPSO is not robust, this metaheuristic is efficient since it requires only 7 iterations to converge to the minimum value. However, this minimum value is higher than that found by other metaheuristics; therefore, this optimization technique presents a fast but premature convergence. On the other hand, GA and EP present a slow convergence process (considered inefficient metaheuristic).

Again, the results of using parallel processing are shown in Table XI, considering 1, 2 and 4 workers with a population of 60 individuals and a maximum iteration number of 500. Table XI shows that all metaheuristics require greater computational time when working with serial processing (1 worker), and

Table VII

Performance of the metaheuristics for the Garver system, scenarios B1 and B2 with 60 individuals.

Scenario	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2	B1	B2
Algorithm	GA		BB-BC		PBILc		EP		CAEP		DE		LPSO		DE-PBILc		CE-PSO		EPSO	
Success Rate (%)	50	70	20	10	70	100	70	100	100	100	100	100	100	90	100	100	40	40	30	100
Eval. F.O	5,903	3,964	600	780	5,511	4,056	4,071	2,454	2,100	2,070	1,393	1,506	2,286	1,800	1,509	2,130	2,842	1,462	11,175	7,236
Stand. Deviat. Iter.	30	28	3	0	12	9	18	8	4	3	5	3	13	12	10	4	51	12	39	12
Average Iter.	97	65	10	13	91	67	67	40	35	34	48	25	38	30	49	36	84	39	58	38
Lowest Cost (M$)	170.63	36,483.4	170.63	36,527.8	170.63	36,483.4	170.63	36,483.4	170.63	36483.4	170.63	36,483.4	170.63	36,483.4	170.63	36,483.4	170.63	36,483.4	170.63	36,483.4
Higher Cost (M$)	269.61	36,497.54	242.34	36,636.61	208.14	36,483.4	202.44	36,483.4	170.63	36,483.4	170.63	36,483.4	170.63	36,489.9	170.63	36,483.4	252.04	36,499.3	234.4	36,483.4
Cost Lines (M$)	170	341	170	261	170	341	170	341	170	341	170	341	170	341	170	341	170	341	170	340
Total Gen. (MW)	784.39	773.16	784.39	778.1	784.39	773.16	784.39	773.16	784.39	773.16	784.39	773.16	784.39	773.16	784.39	773.16	784.39	773.16	784.39	773.16
Gen. Cost (M$)	-	36,142.4	-	36,266.8	-	36,142.4	-	36,142.4	-	36,142.4	-	36,142.4	-	36142.4	-	36,142.4	-	36,142.4	-	36,142.4
Shunt Comp. Cost (M$)	0.63	0	0.63	1.02	0.63	0	0.63	0	0.63	0	0.63	0	0.63	0	0.63	0	0.63	0	0.63	0
Total Shunt Comp. (MVAr)	63.96	0	63.96	102	63.96	0	63.96	0	63.96	0	63.96	0	63.96	0	63.96	0	63.96	0	63.96	0

Table VIII

Expansion plans for the IEEE 24-bus system.

Added lines	Scenarios					
	DC3	DC4	C1	C2	C3	C4
1-2	0	0	0	0	2	0
1-5	0	1	0	0	0	1
4-9	0	0	0	0	1	0
5-10	0	0	0	0	1	0
6-10	1	1	1	1	2	2
7-8	2	3	2	2	4	3
10-11	0	0	0	0	1	0
10-12	1	1	0	1	0	0
11-13	0	0	0	0	1	1
15-21	0	0	0	0	1	0
15-24	0	0	0	0	1	0
14-16	1	1	0	0	0	0
14-23	0	0	0	0	2	1
16-17	0	1	0	0	0	0
20-23	0	1	0	0	0	0
Total Cost (M$)	152	256	48	98	63,162.97	65,667.07
Total Active Power Generation (MW)	8,549.99	8,549.99	8,853.82	8,840.31	8,731.68	8,770.8
Total Active Power Gen. Cost (M$)	-	-	-	-	62,582.97	65,413.07
Lines Cost (M$	152	256	48	98	580	254
Total Shunt Comp. (MVAr)	-	-	1,993.89	2,233.58	1,830.82	1,724.38

Table IX

Success rates of the metaheuristics for IEEE 24-bus system.

Algorithm	GA	BB-BC	PBILc	EP	CAEP	DE	LPSO	DE-PBILc	EPSO	CE-PSO
Scenario	Success Rate (%)									
C1	40	0	70	0	60	90	80	100	70	60
C2	50	0	100	0	80	100	90	100	100	40
C3	30	0	20	0	30	100	30	70	20	0
C4	100	0	80	0	80	90	30	100	90	50

important computing time savings result from using parallel processing.

C. IEEE 118-bus system

The 118-bus system is considered a very challenging test system as far as transmission expansion planning is concerned. It consists of 118 buses, 177 transmission lines, 9 transformers, 54 generators, total active and reactive power demand of 3,733MW and 1,462.98MVAr respectively. The maximum allowed number of circuits per right-of-way is eight. The complete data for the original system can be found in [41]. The data presented in [41] have been modified since the original system (**Z** scenario) does not require the addition of transmission lines (Table XII). Therefore, the line ratings have been reduced to create line congestion in the initial network. The analyzed scenarios of the modified system comprise a mix of dispatchable and non-dispatchable generation. As for scenarios considering operating costs, a capacity factor $CF = 0.6$ was set for all generators. The operating costs ($\$/MWh$) of each generator were set to: $\beta_4 = \beta_6 = \beta_8 = \beta_{15} = \beta_{19} = \beta_{24} = \beta_{27} = \beta_{31} = \beta_{34} = \beta_{40} = \beta_{42} = \beta_{72} = \beta_{73} = \beta_{85} = 0.002$, $\beta_{10} = \beta_{12} = \beta_{25} = \beta_{49} = \beta_{54} = \beta_{69} = \beta_{80} = \beta_{89} = \beta_{92} = \beta_{99} = \beta_{100} = 0.0012$, $\beta_{18} = \beta_{32} = \beta_{36} = \beta_{46} = \beta_{55} = \beta_{56} = \beta_{62} = \beta_{76} = \beta_{77} = \beta_{82} = \beta_{104} = \beta_{105} = \beta_{111} = \beta_{112} = \beta_{113} = 0.017$, $\beta_{91} = \beta_{110} = \beta_{116} = 0.0022$, $\beta_{74} = \beta_{90} = \beta_{103} = \beta_{107} = 0.0037$, $\beta_{26} = \beta_{87} = 0.0010$, $\beta_{59} = \beta_{61} = 0.0013$, $\beta_{65} = \beta_{66} = 0.0008$ and $\beta_{70} = 0.0015$.

In this system, four scenarios were analyzed: *i)* shunt compensation with 1,000$/MVAr at load buses (D1), *ii)* no shunt compensation (D2), and the last two scenarios D3 and D4 are similar to the previous ones but considering the power generation cost. Shunt compensation limits were set to -1,000 and 1,000MVAr. The final plans found for each scenario are shown in Table XII, with a population of 100 individuals, 3,000 iterations allowed and performing 10 tests. Scenario D1 presents a total investment cost of $27.89M, where $27.7M corresponds to the addition of lines and $0.19M to the shunt compensation cost. On the other hand, scenario D2 presents a total investment cost of $45.1M corresponding to the addition of lines. Scenario D1 presents a lower total investment cost as compared to scenario D2 since part of the reactive demand is supplied by the shunt compensators. Scenario D3 presents a total cost of $43,233.002M where $337.8M represents the addition of lines and $42,922.14M corresponds to the system operating cost. Also, the obtained plan require $0.011M per shunt compensation cost. The increase in the cost for added lines (compared to scenario D1) results in important savings during the operation of the system. On the other hand, although the scenario D4 requires a lower investment cost due to the addition of lines ($310M) concerning the scenario D3 ($338.7M), this increase in the cost for added lines results in important savings during the operation of the system.

The metaheuristics that presented the best performance for the previous test systems were considered to solve the IEEE 118-bus system scenarios (PBILc, DE, CAEP, LPSO, EPSO, CE-PSO and DE-PBILc). Table XIII shows the success rates of the metaheuristics for each scenario D, where only the DE-PBILc is robust for almost all scenarios, while the other

Table X

Performance of the metaheuristics for scenarios C1 and C3 (IEEE 24-bus system) with 60 individuals and 500 iterations allowed.

Scenario	C1	C3	C1	C3	C1	C3	C1	C3	C1	C3	C1	C3	C1	C3	C1	C3	C1	C3	C1	C3
Metaheuristic	GA		BB-BC		PBILc		EP		CAEP		DE		LPSO		DE-PBILc		CE-PSO		EPSO	
Success Rate (%)	40	30	10	10	70	20	10	10	60	30	90	100	80	30	100	70	60	10	100	20
Eval. F.O	21,336	21,288	900	480	9,882	9,060	19,980	29,640	15,020	27,520	2,086	11,970	5,565	7,200	2,444	6,960	2,325	3,540	2,257	50,565
Stand. Deviat. Iter.	85	100	0	0	17	12	0	0	97	31	10	23	9	41	6	13	37	0	8	57
Average Iter.	349	349	15	8	164	151	333	494	250	458	95	199	92	120	100	116	68	108	7	260
Lowest Cost (M$)	48	63,162.9	70	63,448.1	48	63,162.9	51	63,168.9	48	63,162.9	48	63,162.9	48	63,162.9	48	63,162.9	48	63,181.1	70	63,162.9
Higher Cost (M$)	78	63,167.6	148	63,873.5	78	63,171.08	134	63,218.5	70	63,167.5	70	63,162.9	70	63,193.2	48	63,163.4	70	62,603.1	70	63,186.8
Cost Lines (M$)	48	580	70	414	48	580	51	649	48	580	48	580	48	580	48	580	48	578	70	580
Total Gen. (MW)	8,853.82	8,731.68	8,851.11	8,755.75	8,853.82	8,731.68	8,853.15	8,724.27	8,853.82	8,731.68	8,853.82	8,731.68	8,853.82	8,731.68	8,853.82	8,731.68	8,853.82	8,732.13	8,851.11	8,731.68
Gen. Cost (M$)	-	62,582.9	-	63,034.1	-	62,582.9	-	62,519.9	-	62,582.9	-	62,582.9	-	62,582.9	-	62,582.9	-	62,586.8	-	62,582.9
Total Shunt Comp. (MVAr)	1,993.89	1,833.21	2,662.65	2,009.18	1,993.89	1,833.21	2,009.26	1,807.83	1,993.89	1,833.21	1,993.89	1,833.21	1,993.89	1,833.21	1,993.89	1,833.21	1,993.89	1,827.87	2,662.65	1,833.21

Table XI

Computing time for IEEE 24-bus system (scenario C1) using parallel processing.

Numbers of workers		Metaheuristic									
		GA	BB-BC	PBILc	EP	CAEP	DE	LPSO	DE-PBILc	CE-PSO	EPSO
1 (Serial)	Time (min)	22.4	24.1	28.5	26.7	25.6	3.5	25.7	4	13.03	94
2		15.3	13.5	14.3	15.8	14.6	2.8	15.7	3.6	9.5	55.4
4		10.1	9.1	8.5	9.4	8.5	2.1	9.2	2.3	4.9	36.6

Table XII

Expansion plans for IEEE 118-bus system.

Added lines	Scenario				
	Z [41]	D1	D2	D3	D4
4-5	0	0	0	0	1
3-5	0	0	0	1	1
8-9	0	1	2	2	2
8-5	0	1	1	1	1
9-10	0	1	1	1	1
15-17	0	0	0	1	1
25-27	0	0	0	1	1
30-17	0	0	0	1	1
26-30	0	0	1	2	2
34-37	0	0	0	1	1
38-37	0	0	1	1	1
63-59	0	0	0	1	1
63-64	0	0	0	1	1
38-65	0	0	0	2	1
64-65	0	0	0	1	1
69-75	0	0	0	1	2
77-78	0	0	0	1	1
82-83	0	0	0	1	1
85-86	0	0	0	2	2
86-87	0	0	0	2	1
80-99	0	0	0	1	1
94-100	0	1	0	1	1
Total Cost (M$)	0	27.89	45.1	43,233.002	43,233.5
Total Active Power Gen. (MW)	6,387.34	6,400.08	6,369.45	6,361.69	6,364.9
Total Active Power Gen. Cost (M$)	-	-	-	42,894.291	42,923.5
Lines Cost (M$)	0	27.7	45.1	338.7	310
Shunt Comp.	No	Yes	No	Yes	No
Shunt Comp. Cost (M$)	-	0.19	-	0.011	-

Table XIII

Success rates of the metaheuristics for IEEE 118-bus system.

Metaheuristics	PBILc	CAEP	DE	LPSO	DE-PBILc	CE-PSO	EPSO
Scenario	Success Rate (%)						
D1	20	0	80	0	90	20	50
D2	0	0	80	0	100	20	100
D3	0	0	0	0	70	0	0
D4	0	0	0	0	60	0	0

metaheuristics (PBILc, DE, CAEP, LPSO, EPSO and CE-PSO) showed lower success rates. As far as the scenarios, a significant reduction in robustness is noted for DE, since it is robust for scenarios the D1 and D2 (success rate $= 80\%$), but significantly reduces its robustness (success rate $= 0\%$) in more complex the scenarios (D3 and D4). Regarding the dimensionality, the most significant reduction in robustness are presented by LPSO and CAEP since those metaheuristics are robust on low dimensionality systems (Garver 6-bus system with the success rates $> 60\%$), but as the dimensionality grows those metaheuristics reduces its robustness (IEEE 118-bus system with the success rates $= 0\%$ for all of the scenarios). Additionally, Table XIII shows that the EPSO is highly robust only for scenario D2 (success rate $= 100\%$) while it loses its robustness significantly for the other scenarios. Even for scenarios D3 and D4, EPSO is not able to find the optimal solution found by the other metaheuristics (success rate $= 0\%$).

The results of the metaheuristic performance are shown in Table XIV. The metaheuristic with the highest efficiency and robustness is DE-PBILc since it allows to find the minimum cost solution in fewer average iterations presenting the highest success rate. Although DE is robust for scenarios the D1 and D2, it is not efficient since it requires 1688 iterations to find the minimum value while DE-PBILc only requires 283 iterations. In addition, DE-PBILc presents a low convergence variability ($20 <$ Standard Deviation). On the other hand, EPSO is efficient and robust only for scenario D2 since it has a success rate of 100 % and it finds the minimum value in 38 iterations (on average). For the other cases, since EPSO is not robust, its efficiency cannot be measured. Fig. 4 shows the convergence process for each metaheuristic where EPSO presents a fast convergence. On the other hand, DE and CAEP present a slow convergence process.

To test the advantages of parallel processing, the computing time was registered using the different metaheuristics (see Table XV), considering 1, 2 and 4 workers with a population of 100 individuals and a maximum iteration number of 3,000, noting that the computing time is significantly lower when the number of workers increases for all metaheuristics.

VI. Summary of the results

The results section showed that:

Table XIV

Metaheuristics performance for the IEEE 118-bus system, with a population of 100 individuals, 3,000 iterations allowed and performing 10 tests.

Scenario	D1						
Metaheuristic	**PBILc**	**CAEP**	**DE**	**LPSO**	**DE-PBIL**	**EPSO**	**CE-PSO**
Success Rate (%)	20	10	80	10	90	50	20
Eval. F.O	62,100	276,800	16,167	22,700	10,790	11,074	4,025
Stand. Deviat. Iter.	0	0	317	0	7	28	15
Average Iter.	621	2,768	1,688.2	227	283	36	70
Lowest Cost (M$)	27.89	427.8	27.89	62.38	27.89	27.89	27.89
Higher Cost (M$)	60.77	495.6	32.09	119.25	32.94	48.5	56.5
Cost Lines (M$)	27.7	426.5	27.7	61.6	27.7	27.7	27.7
Total Gen. (MW)	6,400.08	6,380.4	6,400.08	6,401.8	6,400.08	6,400.08	6,400.08
Gen. Cost (M$)	-	-	-	-	-	-	-

Scenario	D2						
Metaheuristic	**PBILc**	**CAEP**	**DE**	**LPSO**	**DE-PBIL**	**EPSO**	**CE-PSO**
Success Rate (%)	10	10	80	10	100	100	20
Eval. F.O	49,900	267,900	18,887	27,900	12,363	17,478	32,975
Stand. Deviat. Iter.	0	0	239	0	9	31	604
Average Iter.	499	2,679	2,208.2	279	298	38	650
Lowest Cost (M$)	53.2	530.9	45.1	63.1	45.1	45.1	45.1
Higher Cost (M$)	113.9	536.4	49.5	102.7	45.1	45.1	83.7
Cost Lines (M$)	53.2	530.9	45.1	63.1	45.1	45.1	45.1
Total Gen. (MW)	6,374.5	6,357.7	6,369.4	6,368.5	6,369.4	6,369.45	6,369.4
Gen. Cost (M$)	-	-	-	-	-	-	-

Scenario	D3						
Metaheuristic	**PBILc**	**CAEP**	**DE**	**LPSO**	**DE-PBIL**	**EPSO**	**CE-PSO**
Success Rate (%)	10	10	10	10	60	20	10
Eval. F.O	70,200	228,700	172,740	38,500	40,500	289,870	67,485
Stand. Deviat. Iter.	0	0	0	0	20	23	0
Average Iter.	702	2287	2879	385	405	935	1,343
Lowest Cost (M$)	43,241.1	43,602.2	43,294.7	43,255.3	43,233	43,240.8	43,247.03
Higher Cost (M$)	43,290.4	43,648.3	43,740.8	43,273.7	43,235.8	43,265.9	43,391.6
Cost Lines (M$)	285.3	742	359	270.5	338.7	321.4	305.9
Total Gen. (MW)	6,368.5	6,357.7	6,366.4	6,371.7	6,361.6	6,364.1	6,366.6
Gen. Cost (M$)	42,955.8	42,859.8	42,935.7	42,984.8	42,894.2	42,919.4	42,941.1

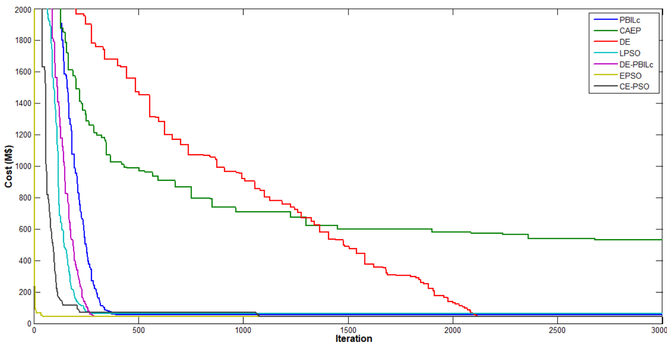

Fig. 4. *Convergence for scenario D2 118-bus system.*

Table XV

Computing time for the IEEE 118-bus system (scenario D1) using parallel processing.

Numbers of workers		**Metaheuristic**						
		PBILc	**CAEP**	**DE**	**LPSO**	**DE-PBILc**	**EPSO**	**CE-PSO**
1 (Serial)	**Time (h)**	15.8	16.6	1.4	16.6	13.52	30.2	13.6
2		9	8.5	1.3	9.7	8.1	20.1	3.5
4		4.8	4.7	1.2	4.9	3.5	10.8	2.1

1) In the Garver 6-bus system, the most metaheuristics are robust (success rate $> 90\%$), efficient (low number of iterations to converge), and low variability of convergence (standard deviation < 12), except GA and BB-BC. While the problem is more complex (Garver with non-dispatchable generation), some metaheuristics reduce their robustness (PBILc, EPSO, and CE-EPSO). On the contrary, CAEP, DE, and DE-PBILc present the highest success rate (100%) and lowest variability of convergence (Standard Deviation < 10) for all scenarios.
2) For the IEEE 24-bus system, only the DE-PBILc and DE metaheuristics maintain acceptable robustness (success rate $> 70\%$) for all the analyzed scenarios, where DE-PBILc presents the lowest variability of convergence (standard deviation < 13) and the highest efficiency (iterations< 116 to converge) if compared to DE (standard deviation < 23 and iterations< 199, respectively).
3) For the most challenging system, the IEEE 118-bus network, only DE-PBILc is robust for all scenarios, presenting as well low variability of convergence and a low number of iterations to converge, if compared to the other metaheuristics. Some metaheuristics such as DE and EPSO are robust only for some cases and the rest of techniques (PBILc, CAEP, LPSO, and CE-EPSO) presents poor robustness for all scenarios.
4) When the TNEP problem, considering both expansion and operation cost, is compared to that considering only lines cost, the final plan obtained presents a higher cost due to the added transmission circuits, but that higher cost is compensated with savings during the operation of the system.
5) As the dimension of the problem increases, the use of parallel processing is very favorable since the saving in computational time is significant.

VII. Conclusion

In this article, a new very robust and efficient metaheuristic, named DE-PBILC, was presented. It is a hybridization between DE and PBILc. Its robustness and efficiency were tested thorough an exhaustive performance comparison against some conventional (GA, LPSO, DE, and EP), and top emerging (BB-BC, EPSO, PBLIc, CAEP, CE-PSO) optimization techniques applied to solve the static TNEP problem using the AC model. Well known test systems with challenging scenarios were used to obtain the results. The most reliable and efficient optimization technique is DE-PBILc since it maintained its robustness and efficiency even for large-scale systems and the most complex scenarios when compared to the other metaheuristics. Therefore, it can be concluded that DE-PBILc is efficient to solve simple (Garver 6-bus system and IEEE 24-bus system) and much more complex and realistic test systems (IEEE 118-bus system). Besides, the proposed TNEP formulation increased the complexity of the problem by incorporating reactive power compensation and the power system operating cost in the objective function. Results in some scenarios showed that an increased investment cost in transmission circuits and/or shunt compensation can be compensated by a smaller operating cost. Additionally, it was proved the computational effort is reduced when the parallel processing is used. Future work may consider the implementation of techniques that improves the performance of metaheuristics algorithms. Also, more complex formulations can be proposed to test the benefits of this new hybrid metaheuristic.

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